Sequential Experiments for Technological Applications: Some Examples

Esperimenti Sequenziali nelle Applicazioni alla Tecnologia: Alcuni Esempi

Daniele Romano
Dept. of Mechanical Engineering, University of Cagliari, Piazza d’Armi, Cagliari, Italy
email: romano@dimeca.unica.it

Riassunto: Dopo una rassegna degli esperimenti sequenziali per applicazioni tecnologiche, il lavoro descrive due contributi dell’autore nel settore della progettazione ingegneristica. Il primo contributo si riferisce alla progettazione robusta di parametri e tolleranze. Il secondo all’integrazione di sperimentazione fisica e simulata per l’innovazione di prodotto. Viene proposta la tesi che gli esperimenti simulati, cioè condotti su codici di calcolo computerizzati, siano un campo di naturale applicazione di approcci e procedure adattativi per i quali è ragionevole attendersi numerosi nuovi sviluppi metodologici.

Keywords: Sequential experimentation, Adaptive design, Computer experiments, Robust Design, Global optimization.

1. What Is Sequential in DoE?

A sequence is “a series of related things or events, or the order in which they follow each other”. Taking this meaning, the wording sequential experiments is ambiguous as it may indicate either a sequence of related experiments or a sequence of related runs within individual experiments. It is also not fully appropriate as it conveys only a piece of information about the whole thing. The shared concept in the twofold use of sequential in DoE is that some decisions concerning the design are not made a-priori - prior to starting the experimental activity thereby basing only on a-priori knowledge - but at one or more times posterior to the start of the experimentation. The advantage is that the delayed decisions are more informed since one is also able to exploit the newly acquired experimental data. This is an adaptive mechanism. As adaptation (“the process of changing something to suit different conditions or uses”) includes the concept of sequence but not vice versa, adaptive would be more significant than sequential. In experimental design adaptation can act run-wise and experiment-wise depending on whether decisions are about the next allocation within an experiment (sequential design) or the next experiment within the whole investigation (sequential experimentation). A well-known technological paradigm of adaptation is the Kalman filter used in control engineering (Maybeck 1979). Optimal on-line control of a linear dynamic system, i.e. best tracking of its output on a desired time pattern, is obtained by continuously

---

1 The paper is financially supported by MIUR within the framework of the PRIN 2005 project “Progettazione statistica dell’innovazione ‘continua’ di prodotto”.
adapting the level of a control signal to the current state of the system as estimated by exploiting a-priori knowledge (the system model) and on-line measurements. This short-term adaptive mechanism is very similar to that used in sequential designs. However, despite of this well-known example and the fact that adaptive designs are claimed to be more efficient than fixed-sample procedures, see, for example, Gosh & Sen (1991) and Baldi et al. (2005), sequential designs are not popular in classical DoE for engineering applications. This is partly because they are more complex to design (they require two criteria to be assessed at each step, a stopping criterion and a next allocation criterion) and to analyze (dependent observations, design variability and unbalance).

On the other hand, sequential experimentation was introduced by Box and Wilson (1951) as the natural scenario for the use of Response Surface Methodology (RSM) which is finalized to the optimization of engineering processes and products. RSM is applied through a sequence of experiments which gradually attain the optimum with a rationale allocation of the experimental effort. The typical sequence envisages a first screening phase where highly fractionated factorials rule out inactive factors; then, more accurate experiments allow to adjust the levels of the active factors in order to identify a small region containing the optimum. A 3-level Response Surface design is run in this region so that an accurate response model can be estimated. Finally, an optimum, or a satisfactory operating sub-region, is found by analytically working on the estimated model. A confirmatory experiment may close the process. Interestingly, the design of each experiment, involving the selection of factors, levels, response, design, experiment size, is adapted here not only to different conditions, i.e. the increasing level of knowledge about the process, but also to different uses.

2. Sequential experiments using computer simulation

A renewed momentum for the use of an adaptive approach to experimentation has been recently generated by the practice of computer experiments (Sacks et al. 1989, Santner et al. 2003) which has been steadily growing in the last two decades. These experiments are run on a computer code implementing a simulation model of a physical system of interest. The main advantage is that the system becomes more “observable” since computer runs are generally easier and cheaper than measurements taken on a physical set-up.

An adaptive approach arises naturally since the computer can both collect data and run a program to decide what to do next. This is particularly attractive in industrial design applications where the goal is system optimization. Prediction at untried points, most useful in the case of expensive simulation, is made by surrogate models, namely statistical interpolators built from the simulated input-output data. Familiar methods for creating surrogate models are RSM and kriging. The latter is a genuinely Bayesian technique where the deterministic response of the computer is regarded as a realization of a Gaussian random process whose correlation function can be shaped in such a way that the response surface is smooth. Prediction is made by a best linear unbiased predictor (BLUP) minimizing the mean squared prediction error, MSPE. The BLUP is a weighted linear combination of the experimental response values and, by construction, interpolates experiment data with no error. For details see Santner et al. (2003). A major problem of kriging is that it assumes a deterministic code. This choice is restrictive and
unpractical. First, in many instances some important input variables are random in the real process. Typical examples are the design of robust engineering systems, where parameters which are non-controllable by the designer (e.g. external temperature, manufacturing errors) act randomly, and the management and control of queuing systems (e.g. production and telecommunication facilities) where arrival and service times (of parts or phone calls) are random. In such cases the simulation code (e.g. a Finite Element code for a new product, (Romano et al. 2004), a discrete event simulator for inventory systems (Bashyam and Fu 1998)) must be random too. The first contribution in the paper addresses a robust design problem. Secondly, the construction of a computer simulator of a complex system has often some degree of uncertainty. It involves several decisions about different modelling options, numerical algorithms, the assignment of value to physical and numerical parameters. Since often there are no clear-cut best decisions, a huge number of computer codes are compatible with the same physical system. Here again random simulation seems preferable (Romano and Vicario 2001). One practical consequence is that the rationale for using standard statistical tools is restored. Thus regression analysis can be safely used for prediction.

A copious literature on sequential designs for computer experiments is available. Crary (2002) discusses G-optimal and I-optimal adaptive designs iteratively minimizing the maximum Mean Squared Prediction Error (MSPE) and the average (Integrated) MSPE respectively. Jones et al. (1998) use a Bayesian approach to response optimization where the next input site is chosen by a heuristic criterion maximizing the expected improvement in the search for an unconstrained optimum. The procedure is driven by the posterior density of the improvement function, conditional on all data available at each step. Variations of this method accommodating for the presence of constraints are proposed by Shonlau et al. (1998) while Williams et al. (2000) also deal with noise variables. These methods generally start with a space-filling design (a Latin Hypercube Sampling design or a maximin design); then, once the sequential procedure is activated, the estimates of the parameters of the correlation function of the Gaussian process are updated by the method of maximum likelihood at each step or, to reduce the computational burden, after a few. Van Beers and Kleijnen (2005) use bootstrap to estimate the prediction variance at untried sites and then choose as next input the one with the maximum prediction variance. Adaptive designs for optimization in the RSM framework are also available that iteratively reduce the design space. Wang et al. (2001) discard regions with large values of the objective function at each modelling-optimization iteration, while Farhang-Mehr and Azarm (2003) and Lin et al. (2004) use maximum entropy as the criterion. Other criteria for successively reducing the design space include move limits (Wujek 1998) and trust region (Alexandrov 1998).

Another genuine reason for using a sequential approach is that computer and physical experiments should be integrated for the purpose of computer model validation, i.e. determining if the computer model provides an adequate representation of the reality, a requisite for the simulation to be of use (Easterling 1999, Bayarri et al. 2002). An easier sub-problem is that of (computer) model calibration. A common framework for calibration is to consider a set of calibration parameters (different from the control inputs) to be assigned a suitable value in the code so that the distance between simulation and field data is minimized over a target input space. This problem has been addressed with a frequentist approach (Park 1991) but more frequently with a Bayesian approach (Bernardo et al. 1992, Craig et al. 1996, Aslett et al. 1998). The latter authors suggest a sequential design approach in the computer experiments, beginning with
values spanning the prior distribution of the calibration parameters and then adding more points to cover the range of their posterior distribution. Design of the physical experiments has not been given much attention, partly because field data are often very few and/or cannot be designed. However the problem of model validation is much more complex than just tuning some calibration inputs. The computer code may be an unacceptable approximation of the physics of the system because the underlying mathematical model is inadequate or too simplified. Moreover numerical algorithms may also be incorrectly chosen. In this case a solely quantitative approach does not seem appropriate. The second case study of the paper proposes a strategy of sequential experimentation where the expert subjective reasoning is an essential ingredient. Interestingly the objectives of system optimization and model validation are fused together.

3. Combined parameter and tolerance design when \textit{nominal is best}

3.1 The problem

This section describes a procedure of sequential experimentation in the context of Robust Design, perhaps the most original contribution of applied statistics to the field of engineering design (Taguchi 1980, Nair ed. 1992). In Taguchi’s view, the overall methodology is deployed in three steps: system design, parameter design and tolerance design. System design is the selection of the rough product configuration which will be subject to detailed design in two subsequent steps. Parameter design is concerned with finding the nominal setting of design parameters so that an optimal balance is attained between two objectives: the product performance attains, on average, a desired target and its random variability, as determined by the uncontrolled variations of environmental variables when the product is used, is minimized. Finally, tolerance design is aimed at the definition of tolerances, i.e. the allowable deviations of design parameters from their nominal settings, as determined by random manufacturing errors, in order to realize a satisfactory balance between product quality (the tighter the tolerances the higher the quality) and manufacturing cost (the looser the tolerances the lower the cost).

While system design relies mainly on the designer expertise, the other two steps require statistically designed experiments. The technical novelty is that both the mean and the variance of the experimental response are of interest. Separating tolerance design from parameter design, albeit reasonable from an engineering viewpoint, falls short of design optimality. In fact, the combined array approach (Myers \textit{et al.} 1992) offers a natural framework for studying design parameters and their tolerance conjointly by including them in the same experiment. However, simultaneous optimal selection of parameters and tolerances is seldom a viable option in practice, as the experimental cost is bound to be excessive.

We describe a procedure for joint parameter and tolerance design (Romano \textit{et al.} 1999) entailing a closely controlled amount of experimental burden in the case when nominal specifications are to be met precisely. This is typical, for example, in the design of measurement devices where lack of bias, i.e. of systematic deviation of the measurement result from the true measurement value, is a primary metrological requirement. The design process is split into three stages. The first covers an experiment run exclusively in the parameter space, a subset of which is thereby identified as
meeting the target specification relative to the mean response. Then an experiment is run in a sampling space determined by the Cartesian product of this subset with the space of noise factors, including both environmental and tolerance factors. The final stage performs the simultaneous synthesis of parameters and tolerances via an optimization problem whose objective function accounts for quality and cost according to a suitable criterion.

3.2 The procedure
Let be a functional characteristic of a product, \( x \in X \) the vector of the design parameters and \( \xi \in \Xi \) the vector of the identifiable noise variables, including tolerances of design parameters, composed of independent normal random variables with zero mean and constant variances \( \sigma_{\xi}^2 \). All unidentified disturbances and the measurement error are pooled in a normal random error \( \varepsilon \) with zero mean and variance \( \sigma_\varepsilon^2 \).

Let us assume for \( y \) the following additive model:

\[
y = \eta(x) + \delta(x; \xi) + \varepsilon
\]

with

\[
\delta(x; \xi = 0) = 0 \quad ; \quad E(\delta) = 0
\]

where \( \eta \) is the deterministic part containing the dependence of \( y \) on the design parameters only plus a constant, and \( \delta \) the stochastic part involving noise variables, alone or in interaction with the design parameters. The essence of the method is to study separately the first two terms on the right hand side of model (1) exploiting suitable \textit{a-priori} characteristics (2) of the model. Requirements (2) are not uncommon; for examples they are met in the dual response method (Myers et. al. 1992). Under the condition \( \xi = 0 \) model (1) reduces to:

\[
y(\xi = 0) = \eta(x) + \varepsilon
\]

Since the target performance must be guaranteed in the absence of noise (\textit{nominal is best}), imposing \( y = y_{\text{nom}} \) in the above equation identifies a subset \( \Gamma \) of the parameter space which spans the optimal parameter setting:

\[
\Gamma \subseteq X \quad / \quad y_{\text{nom}} = \eta(x \in \Gamma; \xi = 0) = \eta_r
\]

By running a first experiment in the parameter space only, with noise variables fixed at zero, the function \( \eta \), and the subset \( \Gamma \) thereafter, can be estimated. Since \( \Gamma \) has a lower dimensionality than the parameter space it can be parameterized using a number of variables less than the number of design parameters. Let \( t \) be the vector of the parameterization variables:

\[
\Gamma : \quad x = x(t)
\]

The stochastic term is then studied only over \( \Gamma \times \Xi \):

\[
\delta_{\Gamma \times \Xi} = \delta(x(t); \xi)
\]
by means of an experiment, with control factors \( t \) and noise factors \( \xi \), whose model is:

\[
y = \eta_{\Gamma \times \Xi} + \delta_{\Gamma \times \Xi} + \epsilon \tag{6}
\]

Since \( \eta \) is a function of the design parameters \( \eta_{\Gamma \times \Xi} = \eta_{\Gamma} \) only, and from Eq. (4) there follows:

\[
y - y_{\text{nom}} = \delta_{\Gamma \times \Xi} + \epsilon \tag{7}
\]

Thus the second experiment allows for the estimate of \( \delta_{\Gamma \times \Xi} \). Recalling that only an estimate of \( \Gamma \) is available we can gain accuracy if \( y_{\text{nom}} \) is replaced in Eq. (7) by the experimental values \( y(x(t_i); \xi=0) \) where \( t_i \) (\( i=1\ldots n_t \)) are the levels of the parameterization vector adopted in the second experiment. This requires to add \( n_t \) extra runs. Finally, turning to in-process conditions where noise factors are random, we can express the mean and the variance of \( y \) in \( \Gamma \times \Xi \):

\[
\mu_y = \eta(x) \\
\sigma_y^2 = \text{Var}_\xi(\delta(x; \xi)) + \gamma(x; \sigma^2_{\xi_1}, \sigma^2_{\xi_2}, \ldots, \sigma^2_{\xi_n}) + \sigma^2_\epsilon \tag{8}
\]

where the function \( \gamma \) in the variance equation can be derived by substituting \( \delta \) with its first-order Taylor’s approximation. We have highlighted that the in-process mean depends only on the design parameters, whereas the in-process variance depends also on the variance of all noise factors.

The optimal setting of parameters and tolerances is finally arrived at by minimizing a cost function somehow involving the in-process mean and variance of the response:

\[
C(\mu_y, \sigma_y^2) = C(x, \sigma^2_{\xi_{\text{tol}_1}}, \sigma^2_{\xi_{\text{tol}_2}}, \ldots, \sigma^2_{\xi_{\text{tol}_m}}) \tag{9}
\]

At the right hand side of Eq. (9), only design parameters and variances of tolerance noise factors are displayed as arguments of the cost function in order to emphasize that their setting is the goal of the integrated parameter and tolerance design. The common choice is to fix the length of the tolerance interval around the nominal specification at \( 6\sigma_{\text{tol}} \). When the problem includes tolerance design, the relationship between each tolerance and the manufacturing cost must be specified as well. In this case the cost function usually sums up two terms expressing quality loss and manufacturing cost respectively.

The method has advantages worth highlighting:

1. Parameter design and tolerance design are accomplished simultaneously.
2. Using two sequential experiments, each one in a subspace (\( \Gamma \) and \( \Gamma \times \Xi \) respectively) of the sampling space corresponding to a unique huge experiment, the experimental load is sizably reduced without sacrificing the quality of the result. Because of the parameterization in the subset \( \Gamma \), parameters are represented by fewer factors in the second experiment (\( t \) has at least one component less than \( x \)). For example, assuming that \( \Gamma \) has only one dimension less than \( X \), the reduction in the number of experimental runs (with \( h+1 \) parameter factors and \( k \) tolerance factors) is given by \( 3^k/(1+3^k) \) (which tends to 3 for \( k \to \infty \)) using 3-level full factorial experiments and
\[(2^{h+k}+2(h+k)+1)/(2^{h+k-1}+2^h+k)\] (which tends to 2 for \(h\) and \(k\to\infty\)) using central composite designs (CCD).

3. The model of the second experiment is peculiar since, by construction, it can neither contain the constant nor any single effect of the parameterization factors. This is because the deviation from the design target (i.e. the response in the second experiment), being constant over \(I\), is unaffected by those factors individually. This allows for either more parsimonious designs or additional degrees of freedom to estimate the experimental error in model (7). The spare degrees of freedom would be \(1+2h+h^2+2b(h,2)+b(h,3)+\ldots+b(h,h)\) (\(b(\cdot,\cdot)\) is the binomial coefficient) for full 3-level factorials and \(1+2h\) for CCDs.

4. Integrated physical and computer experiments

4.1 Description of the problem

In the statistical literature the scope for integrating physical and numerical experiments does not seem to go much beyond model calibration. However, in hi-tech industrial sectors, e.g. aerospace, the combined use of intensive simulation and lab tests is a daily practice. Therefore a more interesting research problem is how to integrate simulated and physical experiments to build knowledge about a real system while dynamically modifying the computer code to get closer and closer approximations to the reality. This problem combines two of the most common objectives of sequential computer experiments: system optimization and model validation. There is also an economic implication. As computer and physical runs differ in cost, efficient allotment of runs is an issue.

A possible approach to the problem is sketched in Figure 4, left. We figure out a sequential strategy where, under the control of the expert, physical and numerical experiments are alternated. Starting with computer experiments we can explore in depth the design space in order to get innovative findings. It is preferable not to start with a physical prototype as manufacturability constraints would confine the designer’s creativity. Then the expert may either design a moderate amount of physical trials to verify the findings or formulate new hypotheses to be assessed with a new cycle of computer runs. The role of physical experiments is twofold. They are not only used for verification purposes but also to drive a dynamic review of the code. The rationale is that the comparison of physical and numerical results should give to the experts hints on how to modify the code in order to improve its ability to simulate the system. The whole process stops when satisfactory innovative results are validated.

Figure 4: Left: a conceptual framework for creating knowledge in engineering design. Right: Path of knowledge building for the design of the climbing robot.
This approach is an extension of the conventional scientific method, an iterative process that combines, at each step, the expert reasoning (deduction) and the direct experience of the phenomenon of interest (induction), represented by the middle and right branches respectively in Figure 4. The proposed procedure adds simulated observations (left branch). Interestingly, these incorporate both induction and deduction. Induction comes from the interrogation of the code at chosen inputs, but the code itself is deduced from formalised models representing the current state of knowledge in a specific technical sector. The key element is the man-computer synergy. Exploration brings unexpected results to the expert who, in turn, activates creative thinking from them. This makes the path towards innovation easier and systematic.

4.2 The design of a climbing robot
The approach is illustrated by an application to the design of a pneumatic climbing robot, realized at the Dept. of Mechanical Engineering of the University of Cagliari, Italy (Manuello et al. 2003, Atzori 2003). The device (Figure 5) must be capable of reaching the top of vertical structures - posts, trees, bridge cables - carrying equipments to make diagnosis and possibly operate on the structures. In order to cling onto the post, the robot exploits a passive stop mechanism of ad-hoc grasping rings. Elimination of a dedicated actuator for clinging makes the robot lighter. The design objective is to make the upward motion of the robot fast and stable regardless of the post surface conditions. The design strategy, inspired by the integrated approach described before, is sketched in Figure 4, right. It is made up of 18 steps, briefly presented hereafter. A broad exploration of the initial design space - 21 factors - is made on the computer model realized using the mechanical simulation package Working Model (step 1 and 2).

Figure 5: Left: the robot prototype. Right: a simplified mechanical model for the robot design.

Figure 6: The surface response marking the boundary between climbing and falling condition.

Six important factors are identified via two screening experiments (a 32-run Plackett-Burman design and a 32-run $2^{14-9}$ fractional factorial) and a response surface estimated (Figure 6) with a 49-run Box-Behnken design. The response is an indicator of the robot ability to climb with the zero level discriminating between climbing (positive) and falling (negative). A precious discovery made at this stage is that, by changing the position of its centre of mass, the robot is also able to descend in a controlled fashion, something beyond the initial design intent. However, this would require a complex and
costly mechanism for realizing a different mass distribution when commuting to the downward motion.

On the basis of these results a physical prototype is realized, its mechanical parameters characterized by static measurements, and finally its ability to climb on a Plexiglas tube proved in a small set of dynamic tests (step 3 and 4). In these tests only two easy-to-change factors are varied. However, a direct comparison with the simulation is done only at step 6 since the measured pre-loading force of the transversal spring of the prototype is outside the range explored at step 2. Here again a cost constraint prevents from adopting a configuration in line with the numerical analysis. Notice that pre-loading force resulted as the most influential in the computer explorative analysis. Had the prototype been realized first, a great deal of information would never have been collected. This justifies the choice to start with a broad computer exploration, almost free of practical constraints. As the feasibility analysis on the prototype has modified the design region, a new cycle of computer experiments is needed to search for a region where the robot is capable to climb with a fast a regular motion on virtually every post surface (step 7 and 8). This is achieved by the sequence of two experiments, a six-factor Box-Behnken and a 4-factor CCD. In the first experiment, three factors do not belong to the influential set identified after the first computer exploration. They are easy-to-change factors replacing as many factors that have a fixed feasible value in the physical prototype. The experiment was extremely instructive for the expert as it revealed several robot behaviours (monotonic climb, non monotonic climb, climb and then fall, no move, no move and then fall, descend in control, fall). Moreover, it provided evidence that when the angle between the locking rings and the post direction approaches 90° the robot is robust to variations of the friction conditions in the ring-post contact. The second experiment aimed at identifying a region where the robot could always climb. Then a confirmatory experiment, a 2³ factorial with 2 repetitions, using the most important factors from previous analysis was run in the lab (step 9 and 10). This confirmed the finding about robustness. As, for feasibility reasons, the design region had been slightly resized, an identical computer experiment followed for model validation (step 11 and 12). Although there was an acceptable agreement, from the comparison of the results the expert was able to learn a lot about the possible physical causes of a slight model inadequacy, see Figure 7 (A).

**Figure 7:** Physical vs numerical: before validation (A); after validation (B) with the optimized design.

**Figure 8:** contour lines of the mean (solid, mm) and the variance (dashed, mm²) of the climbing step length.
This led to two modifications of the computer code: a more accurate definition of the axial stiffness of the robot as a function of its elongation, the simulation of the stick and slip mechanism governing the contact of the two surfaces for a very low relative motion (step 13). Using the revised code for a new computer replication of the physical experiment (step 14) the agreement was improved (the three largest effects on the robot climbing step and the signs of the others coincide). Thus, the $2^3$ factorial was augmented with center and axial runs (step 15 and 16) for the robust design optimization (step 17, Figure 8). The optimum was eventually confirmed by a lab test (step 18, Figure 7 (B)).

5. Discussion

The overview of less than two decades of literature on computer experiments has shown that they are an ideal environment for the use of sequential planning strategies. Although the easiest and most frequent applications concern adaptive sampling for global optimization, there is reason to predict that the full exploitation of computer experiments in technology will stimulate significant research for developing new protocols of sequential experimentation. In fact, the essence of progress in technology is increased complexity and computer experiments are a powerful tool to deal with it, not only because physical investigation can be expensive or even impossible, but also because they allow an extensive exploration of the system from which innovative findings and/or creative thinking of the expert may derive.

There is an inherent relation between experimental complexity and planning horizon in sequential experiment. Run-by-run adaptive design driven by a quantitative criterion is viable (and possibly more efficient) in “simple” situations, like selection of one out of two treatments in clinical trials or optimization of a single response in computer experiments. When experimental complexity increases a one-run planning horizon becomes too myopic and quantitative criteria too restrictive. Complexity has to do with interactive patterns among multiple items and may appear in different forms: many parts, many physical mechanisms, many input variables, many output variables, many objectives, different scales of observation (small/large), different ways of observation (physical, numerical), different level of approximation (rough, detailed). Whenever the interactive patterns are organized as hierarchical relations a sequential approach to investigation is a natural response. Examples are: an output of a system’s part is input to another, some factors are more important than others (active/inactive factors, control factors/noise factors), some output is more important than another (mean was more important than variance in the first example of the paper), an objective comes before another (exploration precedes optimization in RSM), micro-scale phenomena condition macro-scale behaviour (nano/micro hierarchy in nanotechnology), physical observations are more reliable than computer observations (as in the second example of the paper), detailed models are better than rough models (integration of computer experiments on simulators with a different degree of accuracy is found in Osio and Amon (1996), and Qian et al. (2004)). Although research on these topics is still limited (as an example model validation is still in its infancy) sequential approaches for governing complexity with computer experiments are likely to be an important research topic in the future.
References


Bashyam S., and Fu M.C. (1998) Optimization of (s, S) inventory systems with random lead times and a service level constraint, Management Science, 44(12), 243-256.


