ABSTRACT

White light interferometry is routinely used for the reconstruction of the micro geometry of mechanical parts when high precision is needed. By this technique, the surface height of each point of the test piece is measured by estimating the abscissa of maximum interference. Several approaches have been proposed so far to perform the measurement, from the simple identification of the baricentrum of the intensity, to the FFT analysis or the use of geometric phase shifters. These approaches correspond to different setups and analysis algorithms, so that one would like to know which solution is best.

In this paper, some of the most known design solutions have been analyzed in terms of measurement bias and uncertainty. In the first phase of the study some of the most critical components of the setups have been experimentally characterized, then the various configurations have been analyzed by designed computer experiments coupled with Monte Carlo simulations. The most attractive numerical results have been confirmed by a few physical trials.

1. INTRODUCTION

White Light Interferometry (WLI) offers two advantages over the conventional interferometric techniques. First, WLI has a virtually unlimited unambiguous measurement range, whereas conventional interferometric techniques are usually limited to no more than half a wavelength [1]. Second, it allows to perform an optical sectioning of the object being observed due to the short coherence length of the source. Figure 1 shows an interferogram representing the variation in intensity at a given point in the image as a function of \( x \), the optical path difference; whereas the dotted line is the fringe envelope. The evaluation of the interferogram at each pixel of the image provides the information on the surface profile. In fact the abscissa of the maximum modulation, \( x_M \),
corresponding to a null optical path difference, is exactly the surface height (relative to the mirror initial location) at the point of the test piece.

Different approaches for estimating the abscissa of maximum interference are proposed in the literature, with different optical setups, from the analysis of signal using the Fourier Transform, to the computing of the weighted centroid of intensity signal, and the estimation of maximum modulation [2-6]. These approaches can be divided into two classes: the first uses the intensity function (blue solid line in fig. 1), that can be directly obtained by using a basic WLI system (Linnik/Mirau/Michelson interferometer). In the second, the intensity signal is further processed by phase-shifting algorithms [7] to obtain the fringe modulation function (green dotted line in fig. 1). This latter is used to directly estimate the abscissa of maximum. The phase shift can be mainly induced either by using a piezo-electric translator which moves the reference mirror (or the test piece), or by using a geometric phase-shifter (GPS) [8-11], a combination of waveplates (fig. 2).

2. THE DESIGN METHODOLOGY

Robust Design [12, 13] is the methodology applied to the design of the optical profilometer [14]. It is based on the use of information drawn from experiments conducted on a physical prototype of the system under design. The basic idea is to select an optimal setting of the design parameters in such a way to track the system performance on a desired target while minimizing its random variability.

This can be done by studying how both the design parameters and the disturbance variables affect the mean and the random variability of the performance variables. This knowledge is obtained by running designed experiments [15, 16] on the prototype where the experimental factors are the design parameters, the so-called control factors, and the disturbance variables, the noise factors.

However many physical processes are so complex that it is difficult or even impossible to study by conventional experimental methods. In fact the number of experimental factors may be too large to perform a physical experiment or it may be difficult or economically prohibitive to control the noise factors influencing the system in the experiment. These limitations can be overcome by conducting simulated experiments on a reliable computer model of the system. In a computer experiment, noise factors can be not only controlled, by assigning to them a desired value at each run of the simulation code, but even treated as random variables, as they are in the real system. Besides, computer experiments are usually exceedingly cheaper than physical experiments. As today's computing power has significantly increased, fast and accurate simulation models can replace complex physical prototypes in several instances. However, it is important to keep under control the risk of building unrealiable knowledge, due to the possible unfitness of the computer code to carefully reproduce the system's functioning. This can be done through the formulation of sequential experimental protocols integrating numerical and physical experiments. Fig. 3 shows how extensive numerical experimentation can be iteratively combined with a limited physical experimentation. The former is aimed at finding innovative results through an in-depth exploration of the space of design parameters, whereas the latter verifies the results from numerical experiments and/or provides hints for modifying the computer code in order to improve its simulation ability. The process is managed by the designer who may generate new design hypotheses under the stimulus of the findings coming out over time. Although the process can go on virtually "endlessly, in practice it stops when a satisfactory result is achieved.

Following the above process, in this work the most interesting findings drawn from numerical experiments have been validated by a few lab trials on the physical prototype.

3. DESIGN SOLUTIONS

Figure 4 schematically shows the three main design solutions for the profilometer analyzed in this study. Each solution has one hardware setup and two software setups. The hardware setup includes the basic configuration with the GPS device (solution 3, fig. 2) or without it (solutions 1 and 2).

The "Pre-Processing" section contains algorithms to extract intensity modulation from intensity signal. Two of them refer to the hardware setups including GPS (solution 3). In particular for this latter two well-known algorithms with four and five steps have been selected [17-19].
Configuration 2 requires an ad-hoc algorithm to take into account the dynamic variation of modulation during the translation of the mirror. In fact, if the steps are small enough, they can be viewed as part of the phase shifting process, where the active set of images changes dynamically [20, 21]. Unlike the classical phase-shifting algorithms, where for each point in the interferogram the corresponding value of modulation can be considered constant during the whole acquisition time, in this case it is necessary to consider its dynamic variation. The local approximation of the intensity signal used to develop the new algorithm is the following:

$$I(x) = I_0 + (ax^2 + bx + c)\cos(dx + \phi)$$  (1)

where \(a\), \(b\) and \(c\) are the coefficients of the parabolic approximation of the modulation, \(I_0\) is the mean intensity and \(d\) is the conversion factor between \(x\)-coordinate of the reference plane and the phase beam (\(d = 4\pi/\lambda\) for a Michelson interferometer). Note that there is no pre-processing step for configuration 1.

In the “Post-processing” section, two algorithms are used to estimate the abscissa of maximum modulation: the “weighted centroid”, based on the analysis of the intensity signal, and a curve fitting algorithm which uses the intensity modulation signal. In the first one the abscissae of the sampled intensity are “weighted” by the absolute difference between each sampled intensity value \(I_i\) and the mean value \(\bar{I}_0\):

$$x_M = \frac{\sum |I_i - \bar{I}_0| \cdot x_k}{\sum |I_i - \bar{I}_0|}$$  (2)

where \(x_k\) represents the mirror position at the sampled point \(k\). Since this algorithm is strongly influenced by the tails, the portion of signal where the difference between the intensity values and the mean value are low (hence more affected by the noise) and \(x_k\) is far from \(x'_M\)—the abscissae of maximum \(|I_i - \bar{I}_0|\)—are filtered out (red points in fig. 5).

The curve fitting algorithm estimates the abscissa of the maximum by interpolating the modulation points by means of three different functions, a Gaussian, a Parabola and a Sinc^2. A Levenberg-Marquardt non-linear fit procedure is applied for the Gaussian and Sinc^2 curves whereas a simple linear fit is used for the Parabola.

4. GPS CHARACTERIZATION

The GPS behaviour strongly depends on the wavelength value \(\lambda\) of the incident beam; in fact the phase shift induced by the waveplates is a function of \(\lambda\), so that quarter and half waveplates behave precisely as they are intended at the center wavelength only. This asks for a limited band source, but contrasts with the requirement of a reduced optical coherence length to enhance the optical sectioning capability of the instrument (requiring a broad-band incoherent source). Thus the GPS model represents one of the most critical component of the white light profilometer simulator and needs an in depth experimental verification.

The characterization of the GPS using the setup of fig. 2 is not a simple task. Difficulties arise not only from the hardware complexity, but also from the necessity to implement dedicated phase-shifting algorithms which must be able to estimate a non constant modulation [22, 23]. Owing to this, it has been decided to make use of an ellipsometric approach that allows for a phase-shift estimation by indirectly measuring the intensity values recorded by the video camera when the GPS is placed between two polarizers.

The chosen setup, in the case of the \(\lambda/4-\lambda/2-\lambda/4\) configuration, is schematically represented in fig. 6 (a). Beside the GPS, it is composed by two polarizers, a beam expander and an opaline plate. By simultaneously rotating the
axes of polarizers, by a known angle $\alpha$, a sinusoidal intensity variation is observable by varying the orientation of the $\lambda/2$ lamina ($\beta$).

$$I = 4 \cos(\alpha)^4 \sin(\alpha)^2 \sin(2\beta)^2$$  \hspace{1cm} (3)

Figure 7 (a) shows the theoretical behaviour of the intensity for a wavelength of the source of 632.8 nm, the same wavelength at which the wave plates are supposed to work. In fig. 7 (b) the results of the experimental runs conducted at the Experimental Mechanics laboratory of Mechanical Engineering Department are reported. The surfaces have been obtained by manually setting the orientation of both polarizers and then rotating by known amounts the $\lambda/2$ lamina via a stepper-motor. For each recorded image the mean intensity, corresponding to the z-axis in the graph, has been computed. Each curve corresponds to a different orientation $\alpha$. The graph has been normalized using the maximum experimental intensity.

By looking at the two graphs, a good agreement appears between experimental data and the theoretical model. From a quantitative point of view the R-square between the theoretical model and the experimental data is 0.97.
By the way, the numerical simulation (fig. 7c), confirmed by experimental results (fig. 7d), highlights the fact that the GPS behaviour at wavelengths different from the nominal is far from what is expected basing on the achromatic theoretical model of fig. 7(a).

In order to verify the real impact of this problem on the global performance of the interferometer, it has been decided to realize a statistical simulator based on the numerical models developed for performing the comparison described above.

In this way it has been possible to obtain a reliable characterization of the probabilistic distribution of the error introduced by the GPS performed by using a statistical simulator based on the same numerical models involved in the comparison just mentioned.

5. DESCRIPTION OF THE EXPERIMENT

Table 1 summarizes all the experimental factors considered in the study. They are classified according to their type (N: Noise Factor, C: Control Factor) and the design solution they belong to. Control factors have always fixed levels whereas the levels of noise factors are totally randomized according to their distribution.

Four control factors are common to all the setups. One is the sampling step of the mirror. The second is the standard deviation of the random error ($e_x$) in the x-coordinate of the sampled points; this error is due to imperfect displacement of the reference mirror as caused by errors on the piezo-electric translator. Similarly, another control factor is the standard deviation of the random error ($e_{CCD}$) due to the video acquisition system. The last control factor is the filter bandwidth. Random errors $e_x$ and $e_{CCD}$ are noise factors that are randomized in the experiments according to Gaussian distributions. Three other noise factors common to all the setups are: the first sampled point, which accounts for the probability to have a truncated envelope, the intensity modulation and the knot centering, which accounts for the not symmetric sampling (around the maximum point) of the intensity envelope. Since their values are not fully under control during the real measurement process, they are randomized in the simulated process. The last six factors concern configuration 3 (fig. 2): $e_n$, $e_w$, $e_p$ are the noise factors related to the GPS optical system, while the phase-shifting algorithm, the type of GPS and fitting function are control factors. The latter is also used in configuration 2.

The computer model works differently depending on the hardware setup and the type of signal considered. If the setup includes the GPS, it generates directly the modulation, otherwise the intensity signal is generated. In the first case suitable simulated tests have been performed to characterize the error distribution of modulation taking into account both the GPS type and the phase-shifting algorithm.

6. SIMULATION RESULTS

An overall assessment of the design solutions has been performed by means of simple statistical tools, in particular the analysis of measurement bias, namely a systematic deviation of the measurement result from its true value, and the analysis of measurement uncertainty.
Analysis of measurement bias

Absence of measurement bias, namely a systematic deviation of the measurement result from its true value, is ascertained by testing the null hypothesis $\mu_{\text{true}} = 0$ against the alternative $\mu_{\text{true}} \neq 0$ for each experimental setting of the control factors. Using ordinary t-tests the null hypothesis is rejected at a 1- $\alpha$ significance level if

$$|\bar{x}_i| > z_{1-\alpha/2} \frac{S_i}{\sqrt{r}}$$

where subscript $i$ denotes the experimental treatments, $r$ is the number of replications and $z_{1-\alpha/2}$ is the 1- $\alpha/2$ quantile of the standardized Normal distribution; use of this distribution is justified by the high number of replications ($r=10^4$). A very low rejection rate of the tests (the maximum value is 3.27%, with $\alpha = 5\%$) indicates no measurement bias in the measurement result.

Analysis of measurement uncertainty

The standard deviation of the measurement result is commonly used as a basic indicator of measurement uncertainty. It can be evaluated by looking at the plots of the mean effects and the most significant factor×factor interactions for the three design configurations. As the same scale is used in all the plots, effects can be easily compared across the configurations.

Notice that only one level of the mirror displacement is considered for the second design scheme. This is due to the numerical instability of the dynamic phase-shifting algorithm when larger mirror displacement are used. Schemes 1 and 3 are not affected by this problem and can be used with any mirror displacement.

Looking at fig. 8 it is quite evident that solution 1 (weighted centroid) and 3 (GPS) give a comparable performance, the latter being only slightly better.

Lower uncertainty is obtained by using small mirror steps and CCD with medium-to-high quality. Filter 3 is surely preferable because it significantly reduces the uncertainty across all configurations.

Configuration 2 is characterized by larger main effects. In principle, this is not necessarily bad as it means that the designer can exert more control on the design. Uncertainty is quite sensitive to the error of the mirror displacement and at a higher extent, to the CCD error. The same is true for configuration 1. Therefore, low uncertainty can only be achieved with sacrifice of product cost. However filter 3 is the key component; it makes the device robust against random variations in the CCD errors, for both schemes 1 and 2 and against random variations in the mirror displacement only for scheme 2. Thus solutions with high quality and low cost are feasible.

The last configuration exhibits the lowest uncertainty. Its sensitivity to the phase-shifting algorithm is very low (the 5-image algorithm is slightly better), while with respect to the hardware configuration, the GPS 2 (\(\lambda/4-P\)) is preferable because is more accurate and less expensive. This result is in agreement with what literature suggests [24]. For configuration 3 no particular interaction effect is worth noting.

It should be noted that simulations have been performed considering the same number of images, thereby for solution 1 the mirror step has been reduced because solution 3 needs 4-5 images for each “experimental” point.

![Figure 8 Main effects plots for the three configurations. Standard deviation of the measurement result is used as measurement uncertainty.](image-url)

7. EXPERIMENTAL RESULTS

The results of the computer simulation have been validated by an experimental campaign. Instead of using an alternative, more accurate experimental technique to estimate the standard deviation of the instrument, we opted for a statistical approach: by repeating several times the same measurement, we are able to estimate mean and
variance of the reconstruction. Since the results depend both on the hardware and on the reconstruction algorithm, by changing these parameters it is possible to verify if the main effects shown by the numerical model are experimentally confirmed.

![Figure 9](image1)

**Figure 9** Three images acquired by the 12bit CCD Camera at different mirror positions: (a) 12.3 μm, (b) 16.9 μm, (c) 23.8 μm

Figures 9a–c show three steps of a acquisition sequence and fig. 10 the complete history of a pixel (both simulated and experimental). The sampling step was 150 nm, with a source band of 40 nm centred at $\lambda = 600$ nm and a carrier wavelength of $592$ nm (obtained by fitting the experimental data with the product of a gaussian by a sine function); the camera was a 12 bit CCD sensor (1624x1236) and the piezo translator was monitored using a custom interferometric sensor. The standard deviation obtained by repeating 100 times the measurement and using the weighted centroid algorithm was $702$ nm in the simulator, which closely matches the experimental value ($694$ nm). Note that the interaction between the sampling step and the carrier wavelength badly affects both the experimental and the simulated standard deviation. In fact by reducing (or enlarging) the sampling step of 20 nm shortens the standard deviation of an order of magnitude.

![Figure 10](image2)

**Figure 10** Intensity history of a pixel. Left: experimental; right: simulated

8. CONCLUSIONS

The use of the computer experiment combined with robust design allowed the development of a reliable design tool for white light interferometry. By combining the theoretical study and the experimental validation of the most critical components of the instrument led to the implementation of a robust simulator of the measurement process. Although its validation is not fully completed, the first experimental results show a good agreement with its simulated results.

REFERENCES


