

EQUAZIONI DIFFERENZIALI E SISTEMI

Esercizi proposti

1. Risolvere i seguenti sistemi di equazioni differenziali :

$$\begin{array}{ll} \text{(a)} & \begin{cases} x'_1 = x_1 \\ x'_2 = x_1 + x_2 \end{cases} & \begin{bmatrix} c_1 e^t \\ (c_1 t + c_2) e^t \end{bmatrix} \\ \text{(b)} & \begin{cases} x'_1 = 2x_2 \\ x'_2 = 2x_1 \end{cases} & \begin{bmatrix} c_1 e^{2t} + c_2 e^{-2t} \\ c_1 e^{2t} - c_2 e^{-2t} \end{bmatrix} \\ \text{(c)} & \begin{cases} x'_1 = 5x_1 + 3x_2 \\ x'_2 = -3x_1 - x_2 \end{cases} & \begin{bmatrix} (c_1 + c_2 t) e^{2t} \\ (-c_1 + \frac{1}{3} c_2 - t c_2) e^{2t} \end{bmatrix} \end{array}$$

2. Risolvere i seguenti problemi di Cauchy:

$$\begin{array}{ll} \text{(a)} & \begin{cases} x'_1 = 2x_1 - 4x_2 \\ x'_2 = x_1 - 2x_2 \end{cases} \quad \text{con} \quad \begin{cases} x_1(1) = 1 \\ x_2(1) = -1 \end{cases} & \begin{bmatrix} 6t - 5 \\ 3t - 4 \end{bmatrix} \\ \text{(b)} & \begin{cases} x'_1 = x_1 + x_2 - x_3 \\ x'_2 = x_1 + 2x_2 \\ x'_3 = 2x_1 + 3x_2 \end{cases} \quad \text{con} \quad \begin{cases} x_1(0) = 2 \\ x_2(0) = 0 \\ x_3(0) = 0 \end{cases} & \begin{bmatrix} (2 - t^2) e^t \\ (2t + t^2) e^t \\ (4t + t^2) e^t \end{bmatrix} \\ \text{(c)} & \begin{cases} x'_1 = x_2 \\ x'_2 = x_3 \\ x'_3 = -2x_1 + x_2 + 2x_3 \end{cases} \quad \text{con} \quad \begin{cases} x_1(0) = 0 \\ x_2(0) = 2 \\ x_3(0) = 0 \end{cases} & \begin{bmatrix} e^t - e^{-t} \\ e^t + e^{-t} \\ e^t - e^{-t} \end{bmatrix} \end{array}$$