Wasserstein geometry of the Gaussian model

Giovanni Pistone de Castro Statistics - Collegio Carlo Alberto

This talk is based on joint work with Luigi Malagò (RIST, Cluj-Napoca) and Luigi Montrucchio (Collegio Carlo Alberto)

Abstract

Given two vector Gaussian variables $X \sim N_d(\mu_1, \Sigma_1)$ and $Y \sim N_d(\mu_2, \Sigma_2)$ we consider all possible jointly distributed Gaussian (X, Y) with the given marginals. We define the Wasserstein distance by $d_W((\mu_1, \Sigma_1), (\mu_2, \Sigma_2)) = \inf E[|X - Y|^2]$. It is possible to explicitely compute such a distance and to contruct the corresponding geodesics. It turns out that the Wasserstein distance is a Riemannian distance i.e., there exists a Riemannian metric whose distance is the Wasserstein distance. This gives rise to a true geometric setting, where flows of vector field can be studied. Some interesting problems could be studied in this set-up: optimization; the definition and computations of means; the relation with other geometries (Fisher-Rao and Hessian); submanifolds i.e. parsimonious Gaussian models; second order geometry.