Nonparametric Union-Intersection Approach in Multivariate Permutation Tests

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Abstract: The Union-Itersection (UI) principle for multivariate testing has quite a long story since S.N. Roy (1953). This approach assumes that the hypotheses H_0 and H_1 can be equivalently written as

$$H_0 \equiv \bigcap_{k=1}^{K} H_{0k} \quad \text{and} \quad H_1 \equiv \bigcup_{k=1}^{K} H_{1k},$$

and that the (global) test can be carried out by considering a suitable list of partial tests T_k, each specific for H_{0k} versus $H_{1k}, k = 1, \ldots, K$.

The provided UI solutions are often asymptotically coincident with those obtained by standard likelihood techniques, but due to the great difficulty to parametrically taking account of underlying dependences, very few problems have found explicit parametric solutions.

A proposal, however, which is far from being easy to achieve if one wishes to manage that dependence (which can be much more complex than linear) by using suitable estimators of all coefficients, the number and kind of which are typically unknown. It has a general solution when it is possible to handle such a dependence in a nonparametric way. This goal is obtained within the testing principle based on conditioning on the whole data set. A principle which, when in the gobal H_0 *the whole data set* $\mathbf{X}$ *is sufficient for the underlying distribution* F, provides for exact permutation (i.e. conditional) solutions even in multivariate settings. The related methods are based on the so-called "*NonParametric Combination* (NPC) *of several dependent permutation tests*".

The main goal of present talk is discussing on main properties of the UI-NPC methodology and presenting some of its multivariate applications.