Understanding the confounding structure of factorial experiments

Ulrike Grömping
Beuth University of Applied Sciences, Berlin

Confounding in regular 2-level fractional factorial plans is measured with the well-known concepts of resolution and – more generally – minimum aberration. Xu and Wu (2001) introduced Generalized Minimum Aberration (GMA), which is a natural extension of minimum aberration (MA) to non-regular and also mixed level factorial plans. It is based on the generalized word length pattern (GWLP). The original definition of the GWLP was very technical. Grömping and Xu (2014) derived a simple interpretation of the number of shortest words, which relates the number of shortest words to confounding of main effect model matrix columns by interaction model matrix columns in a coding invariant manner. Coding invariance is of course important for assessing the usefulness of an experimental plan for qualitative factors.
According to Grömping and Xu (2014), the number of shortest words can be interpreted as a sum of R^2 values over orthogonal main effect model matrix columns or – more generally – as a sum of squared canonical correlations between a main effect model matrix and an interaction model matrix. Based on this interpretation, it is possible to dissolve the problem pointed out by Wu and Zhang (1993) of adequate treatment of factors at different numbers of levels in mixed level designs: Grömping (2017) proposes the use of average R^2 frequency tables and squared canonical correlation frequency tables (ARFTs and SCFTs) for detailed investigation of the worst-case confounding. The talk presents these concepts and illustrates their meaning with the help of mosaic plots (Hartigan and Kleiner 1981, 1984), which the author considers a very useful tool for visualizing orthogonal array projections. All methods discussed in this talk are implemented in the R package **DoE.base** (Grömping 2009-2017).

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