Optimization under uncertainty: modeling and solution methods
A Ph.D. level course given within the framework of the Ph.D. program in Mathematics for Engineering Sciences

LECTURER
Prof. Paolo Brandimarte
Dipartimento di Scienze Matematiche
Politecnico di Torino
e-mail: paolo.brandimarte@polito.it
URL: http://staff.polito.it/paolo.brandimarte

AIM
The aim of the course is to strengthen the knowledge of optimization methods, extending modeling and solution procedures to cases affected by significant uncertainty. Uncertainty is pervasive in many branches of engineering and social sciences, including finance, supply chain management, energy markets, and telecommunication networks. Emphasis is on stochastic programming models, but, since a stochastic characterization of uncertainty is not always available, reliable, or appropriate, we will also consider robust optimization frameworks. Furthermore, since solving multistage stochastic optimization models is quite challenging, we will also deal with Approximate Dynamic Programming methods that, among other things, illustrate the connection between mathematical optimization and machine learning. Case studies and examples are used throughout the course to illustrate the relevance of its content.

Audience: the course is given within the framework of our Ph.D. program in Mathematics for Engineering Sciences, but it should also appeal to other Ph.D. students dealing with decision making under uncertainty (e.g., Telecommunications, Energy Planning, etc.).
Prerequisites: some familiarity with standard linear programming models; essentials of probability theory.

CONTENTS
The course consists of eight lectures (one 4-hour, seven 3-hour), for a total of 25 lecture hours (5 credits).

Lecture 1. Optimization modeling under uncertainty.
- Introductory examples: production planning with assembly-to-order under demand uncertainty; chemical production with uncertain quality of raw materials.
- Alternative frameworks: stochastic programming with recourse, chance-constrained optimization, robust optimization.
- Measuring the impact of uncertainty: expected value of perfect information and value of the stochastic solution.
- Modeling multistage problems.

Lecture 2. Theoretical optimization background (4 hours).
- A few bits of convex analysis.
- Solution methods for linear, nonlinear, and mixed-integer programming models.
- Duality in optimization.
- Convexity properties of stochastic programs.

Lecture 3. Decomposition methods for stochastic programming models with recourse.
- L-shaped decomposition.
- Progressive hedging.
- Interior point methods.

- Strengthening mixed-integer linear programs.
- Heuristic methods for stochastic mixed-integer linear programming.
- The role of terminal conditions and costs in dynamic optimization: examples from production planning, pension fund management, and fisheries management.

Lecture 5. Scenario generation.
- Monte Carlo based scenario generation.
- Variance reduction strategies.
• Deterministic methods from numerical integration: Gaussian quadrature and low discrepancy sequences.
• Optimal scenario reduction.

Lecture 6. Risk measurement and risk management.
• Representing risk aversion: utility functions and regret.
• Quantile-based risk measures (VaR, CVaR).
• Trading off profit/return and risk: mean-risk models and efficient frontiers.
• Examples from finance and design of supply networks.

Lecture 7. Robust optimization.
• Non-stochastic representation of uncertainty.
• Different concepts of robustness.
• Solution methods by convex optimization techniques.

Lecture 8. Approximate dynamic programming.
• Dynamic programming and the Bellman equation.
• Learning the value function by Monte Carlo simulation and linear regression.
• Applications: financial option pricing and ambulance deployment.

References
  http://www2.isye.gatech.edu/people/faculty/Alex_Shapiro/SPbook.pdf

COURSE MATERIAL AND UPDATES
Slides will be posted on the Web page of the Probability and Statistics group:
http://calvino.polito.it/~probstat/doctorate.html
If you are interested in the course, I suggest to send me an email message, so that I can provide you with them in advance. The course mailing list will also be used to communicate updates in case of need.

SCHEDULE
Lectures will be given at Dipartimento di Scienze Matematiche (DISMA), Politecnico di Torino, in Aula Buzano (the internal lecture/seminar room of DISMA, third floor).

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