

# Infiltration of a Polymerizing Resin in a Deformable Preform for Fiber Reinforced Composites

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## 1 Introduction

Many fiber reinforced composites are manufactured using industrial processes, named Resin Transfer Moulding, Structural Resin Injection Moulding or Squeeze Casting which consist in injecting the reacting polymer into a porous preform of reinforcing elements. At the end of the production cycle, the liquid matrix, which should uniformly fill up the whole preform, solidifies holding the reinforcing elements together and enabling the transfer of major stresses and loads to the solid preform.

It appears from the literature that the identification of the production cycle parameters to be used in practice is still more an art than a science (see, for instance, [1], [2]) and it has to be developed every times on a “trial-and-error” basis. On the other hand, from an industrial point of view it is important to have a model which allows to monitor the thermal state and the deformations of the solid preform in order to quantify the inhomogeneous characteristics of the final product and reveal in advance the possibility of damages in the reinforcing network, which may even lead to material failure.

For instance, in some particular situations, the pressure driving the flow is large enough to significantly compress the reinforcing network (especially ahead of the advancing infiltration front) altering the preform permeability [3]–[5].

This paper reviews a model, presented in [6], aimed at simulating non-isothermal injection moulding processes, always allowing deformation of the solid preform during the application of pressure and temperature cycle. In addition, while infiltrating resin may undergo an exothermic chemical reaction, usually referred in the literature as curing cycle.

The problem presents the formation of three time-dependent domains, the first one occupied by the liquid only, the second one by the solid preform wet by the

infiltrating the resin and the third one consisting of the uninfiltrated region. It is assumed that sharp fronts divide the three domains.

The mathematical problem originated by the model is essentially a free boundary problem. It considers the whole coupled non-isothermal flow and deformation model, and is constituted by a set of parabolic and hyperbolic nonlinear equations defined in time-varying domains.

## 2 The Mathematical Model

In this section we consider the general 3D non-isothermal infiltration of a resin undergoing to a polymerization process through a deformable porous medium.

Taking as  $t = 0$  the instant at which the porous medium starts to be infiltrated, one can identify two time-varying domains  $D^w$  and  $D^d$  corresponding respectively to the part of the solid preform wet by the infiltrating resin and the one not yet reached by the liquid. If capillary phenomena can be neglected, as in the most production processes,  $D^w$  and  $D^d$  are divided by a sharp interface  $\sigma^i$  which represents the infiltration front.

The mathematical model consists in writing the evolution equations for the state variables in both the wet and the dry region and the equations for the interfaces.

### *Wet Region*

The dependent variables describing the state of the system are:

- $\phi$  volume fraction occupied by the solid constituent. Assuming saturation the volume fraction occupied by the liquid is  $1 - \phi$ .
- $\vec{v}_s, \vec{v}_l$  velocities of the solid and liquid constituents.
- $P$  pore liquid pressure.
- $\theta$  temperature of the mixture. We assume that solid and liquid are locally in thermal equilibrium.
- $\delta$  degree of cure of the resin. It is defined as the ratio between the amount of heat released by the polymerization reaction and the total heat of reaction. So  $\delta$  is a dimensionless parameter ranging between 0 and 1.

Therefore, referring to [6]–[8] for further details, the mathematical model can be based on:

- Mass conservation of solid and liquid constituents

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \vec{v}_s) = 0, \quad (1)$$

$$\frac{\partial \phi}{\partial t} - \nabla \cdot [(1 - \phi) \vec{v}_l] = 0, \quad (2)$$

where we have assumed that the densities of the solid  $\rho_s$  and liquid  $\rho_l$  are constant.

- Stress equilibrium equation for the mixture as a whole

$$\nabla P - \nabla \cdot \mathbf{T}_m = 0, \quad (3)$$

where  $\mathbf{T}_m$  is called excess stress. We remark that in writing (3) we have implicitly neglected body forces and inertial terms. Both are usually very small. In particular, in previous papers [9]–[11] which considered inertia, it was noticed that the dimensionless form of the equations present in front of the inertial term a coefficient which is usually negligible. Actually, it was shown that inertia plays an important role only during the very initial stages of infiltration.

- Darcy's law

$$(1 - \phi) (\vec{v}_l - \vec{v}_s) = -\frac{\mathbf{K}(\mathbf{F}_s)}{\mu} \nabla P, \quad (4)$$

where  $\mathbf{K}$  is the permeability tensor which depends on the deformation gradient of the solid constituent  $\mathbf{F}_s$  and  $\mu$  is the viscosity of the resin. Since we are considering the non-isothermal process with resin cure, the viscosity  $\mu$  depends on both the temperature  $\theta$  and the degree of cure  $\delta$

$$\mu = \mu(\theta, \delta). \quad (5)$$

- Degree of cure of the resin

$$\frac{\partial \delta}{\partial t} + \vec{v}_l \cdot \nabla \delta = f(\theta, \delta), \quad (6)$$

where  $f$  is an experimentally determined function describing the reaction.

- Energy balance for the mixture

$$\begin{aligned} \rho_m C_m \left( \frac{\partial \theta}{\partial t} + \vec{v}_m \cdot \nabla \theta \right) &= \nabla \cdot (\mathbf{k}_m \nabla \theta) + \frac{1}{\mu} \mathbf{K} \nabla P \cdot \nabla P + \\ & (1 - \phi) H_c f(\theta, \delta) - \frac{\rho_s \rho_l \phi (1 - \phi)}{\rho_m} (C_l - C_s) (\vec{v}_l - \vec{v}_s) \cdot \nabla \theta, \end{aligned} \quad (7)$$

where  $\rho_m = \phi \rho_s + (1 - \phi) \rho_l$  is the mixture density;  $C_m$  is the specific heat of the mixture

$$C_m = \frac{\rho_s \phi C_s + \rho_l (1 - \phi) C_l}{\rho_m}, \quad (8)$$

being  $C_s$  and  $C_l$  the specific heats of the solid and liquid;  $\vec{v}_m$  is the mass average velocity

$$\vec{v}_m = \frac{\rho_s \phi \vec{v}_s + \rho_l (1 - \phi) \vec{v}_l}{\rho_m}, \quad (9)$$

$\mathbf{k}_m$  is the thermal conductivity tensor of the mixture as a whole which is modeled as

$$\mathbf{k}_m = \phi \mathbf{k}_s + (1 - \phi) \mathbf{k}_l, \quad (10)$$

being  $\mathbf{k}_s$  and  $\mathbf{k}_l$  the heat conductivities of the solid and liquid;  $\mu^{-1} \mathbf{K} \nabla P \cdot \nabla P$  is the heat produced by the internal friction between solid and liquid;  $(1 - \phi) H_c f(\theta, \delta)$  represents the heat supplied by the exothermic curing reaction of the resin ( $H_c$  is the total heat of reaction) and the last term relates the changes of heat capacity due to infiltration.

### *Dry Region*

In the dry region, one can proceed in a similar way exploiting the fact that the air density and viscosity are very small. First of all, since the air is easily expelled from the preform we can assume that its pressure is constantly equal to the atmospheric value. Moreover the gas contribution to the global stress may be neglected. Finally, the average velocity is equal, in practice, to the velocity of the solid constituent, the composite density is  $\rho_m \approx \phi \rho_s$  and, of course, there is no reaction since no resin is present in the dry region.

Summarizing all these assumptions one has that the state variable in dry region are:

- $\phi^d$  solid volume fraction.
- $\vec{v}_s^d$  solid velocity.
- $\theta^d$  temperature.

The fundamental equations are, respectively, mass, momentum and energy balance

$$\frac{\partial \phi^d}{\partial t} + \nabla \cdot (\phi^d \vec{v}_s^d) = 0, \quad (11)$$

$$\nabla \cdot \mathbf{T}_s = 0, \quad (12)$$

$$\rho_s C_s \left( \frac{\partial \theta^d}{\partial t} + \vec{v}_s^d \cdot \nabla \theta^d \right) = \nabla \cdot (\mathbf{k}_s \nabla \theta^d), \quad (13)$$

where  $\mathbf{T}_s$  is the stress tensor of the dry solid.

To close the model we still have to specify the constitutive equations for the stress tensors  $\mathbf{T}_m$  and  $\mathbf{T}_s$ . In the following we will assume that the wet and the dry solid behave elastically but with different stress–strain relations, as experimentally determined.

### *Interface and Boundary Conditions*

The infiltration problem we are dealing with is, of course, a free boundary problem. One free boundary is the interface  $\sigma^i$  between  $D^w$  and  $D^d$ . Let it be given by the

surface  $\psi_i(\vec{x}, t) = 0$ . As  $\sigma^i$  is a material surface for the liquid its evolution equation is

$$\frac{\partial \psi_i}{\partial t} + \vec{v}_l(\sigma^i) \cdot \nabla \psi_i = 0, \quad (14)$$

being  $\vec{v}_l(\sigma^i)$  the liquid velocity on  $\sigma^i$ .

The other free boundary is the contact surface  $\sigma^e$  between the pure liquid and the wet solid. Let it be given by  $\psi_e(\vec{x}, t) = 0$ . Since the resin, pushed by the pressure gradient, penetrates the porous solid through  $\sigma^e$ , this is a material surface for the solid and therefore its evolution equation is

$$\frac{\partial \psi_e}{\partial t} + \vec{v}_s(\sigma^e) \cdot \nabla \psi_e = 0, \quad (15)$$

where  $\vec{v}_s(\sigma^e)$  is the solid velocity on  $\sigma^e$ .

Following [12] the jump conditions to be imposed are

$$\llbracket \rho_m (\vec{v}_m - \vec{v}_\sigma) \rrbracket \cdot \vec{n}_\sigma = 0, \quad (16)$$

$$\llbracket -P\mathbf{I} + \mathbf{T} \rrbracket \vec{n}_\sigma = 0, \quad (17)$$

$$\llbracket \rho_s \phi \theta C_s (\vec{v}_s - \vec{v}_\sigma) \cdot \vec{n}_\sigma + \rho_l (1 - \phi) \theta C_l (\vec{v}_l - \vec{v}_\sigma) \cdot \vec{n}_\sigma \rrbracket = \llbracket \mathbf{k}_m \nabla \theta \rrbracket \cdot \vec{n}_\sigma, \quad (18)$$

where  $\llbracket \cdot \rrbracket$  denotes the jump across the surface  $\sigma$ ,  $\vec{v}_\sigma$  and  $\vec{n}_\sigma$  are respectively the surface velocity and the normal outside  $D^w$ . In addition, as usual, we assume

$$\llbracket \theta \rrbracket = 0. \quad (19)$$

As the surface  $\sigma^i$  is a material surface for the liquid, following [12] we have

$$\llbracket P \rrbracket = 0, \quad (20)$$

so that (17) rewrites

$$\llbracket \mathbf{T} \rrbracket \vec{n}_\sigma = 0. \quad (21)$$

Let us consider now the boundary condition on  $\sigma^e$ . If the superscript *liquid* denotes the quantities evaluated in the pure liquid region we have

$$\begin{aligned} \phi^{liquid} &= 0, & \vec{v}_l^{liquid} &= \vec{v}_{in}, \\ \mathbf{T}^{liquid} &= 0, & P^{liquid} &= P_o, \end{aligned}$$

being  $\vec{v}_{in}$  the inflow velocity of the resin and  $P_o$  the pressure driving the flow.

From (17) and (20) we get the following conditions

$$P = P_o, \quad (22)$$

$$\mathbf{T}_m \vec{n}_{\sigma^e} = 0. \quad (23)$$

Assuming a known thermal field in the pure liquid region, on  $\sigma^e$  the only condition which we impose on the temperature is equation (19) which rewrites as

$$\theta = \theta_{in}, \quad (24)$$

being  $\theta_{in}$  the temperature of the infiltrating liquid.

The boundary condition to impose to the hyperbolic curing equation is

$$\delta(\sigma^e) = \delta_{in}, \quad (25)$$

on that part of  $\sigma^e$  where the liquid enters the preform, i.e. where  $(\vec{v}_l - \vec{v}_s) \cdot \vec{n}_{\sigma^e} < 0$ .

### 3 1D Non-Isothermal Injection with Curing

Consider, now, the one-dimensional infiltration problem along a principal direction of the permeability tensor and denote by  $x_e(t)$  the left border of the preform and by  $x_f(t)$  the infiltration front. A fully draining condition constraints the right border of the solid preform to be fixed at  $x = L$  but allows, at the same time, both air and liquid to pass through with no resistance.

For  $t < 0$  the whole preform is dry, at rest and undeformed i.e.

$$\begin{aligned} \phi(x, 0) &= \phi_r, & x &\in [0, L], \\ x_f(0) &= x_e(0) = 0. \end{aligned} \quad (26)$$

The liquid matrix flows in the positive direction forced by the pressure gradient and, at  $t = 0$  touches the left border of the preform. The incoming liquid suddenly compresses the solid and then the fluid starts infiltrating. Therefore, while the right border of the preform stays fixed at  $x = L$  the other one moves to  $x = x_e(t)$  and part of the solid, precisely up to  $x = x_f(t)$ , wets up. One then has that the wet and the dry regions are  $D^w(t) = (x_e(t), x_f(t))$  and  $D^d(t) = (x_f(t), L)$  and the problem is characterized by the presence of two free boundaries.

Denoting respectively by  $\tau_m$  and  $\tau_s$  the  $xx$  component of  $\mathbf{T}_m$  and  $\mathbf{T}_s$ , counted as positive in compression, the elasticity assumption means that  $\tau_m$  and  $\tau_s$  are functions of the  $xx$  component of the deformation gradient of the solid which, in one dimension, is

$$(\mathbf{F}_s)_{xx} = \frac{\phi_o}{\phi}, \quad (27)$$

where  $\phi_o$  is the volume ratio in the reference configuration. Hence

$$\tau_m = -\Sigma_m(\phi), \quad (28)$$

$$\tau_s = -\Sigma_s(\phi). \quad (29)$$

In particular, considering as porous solid the network of glass fiber used in the experiment presented in [4], we have assumed

$$\Sigma_m(\phi) = 0.09 [e^{26.4\phi} - e^{26.4\phi_o}] Pa, \quad (30)$$

$$\Sigma_s(\phi) = 0.3 [e^{25\phi} - e^{25\phi_o}] Pa. \quad (31)$$

Following [5] we take the permeability tensor as given by

$$(\mathbf{K})_{xx} = K(\phi) = 10^{-9} e^{-16(\phi-0.6)} m^2. \quad (32)$$

For the remaining parameters several models have been proposed (see [8] for a review). We consider the following

$$\mu(\delta, \theta) = \begin{cases} \bar{\mu} \epsilon^{E_\mu / R\theta} \left( \frac{\delta_g}{\delta_g - \delta} \right)^{c_\mu + d_\mu \delta} & \text{if } \delta < \delta_g; \\ \infty & \text{otherwise;} \end{cases} \quad (33)$$

$$f_c(\theta, \delta) = \left[ c_1 \exp \left\{ -\frac{E_1}{R\theta} \right\} + c_2 \exp \left\{ -\frac{E_2}{R\theta} \right\} \delta^{m_c} \right] (1 - \delta)^{n_c}, \quad (34)$$

where  $\delta_g$  is the resin gel threshold,  $m_c$  and  $n_c$  describe the order of reaction,  $c_1$  and  $c_2$  are the reaction rate constants,  $E_\mu$ ,  $E_1$  and  $E_2$  are activation energies,  $R$  is the gas constant and  $\bar{\mu}$ ,  $c_\mu$  and  $d_\mu$  are constants.

The whole infiltration problem consists, therefore, in solving in  $D^w$  and in  $D^d$  the system of equations (1)–(4), (6), (7) and (11)–(13) coupled with suitable conditions specified on  $x_f$  and the evolution equations for the free boundaries  $x_e(t)$  and  $x_f(t)$ .

Focusing on the dry region one can observe that the elasticity assumptions (28), (29) and the momentum equation  $\partial \tau_s / \partial x = 0$  imply that the volume ratio  $\phi^d$  is space independent

$$\phi^d(x, t) = \phi^d(t) \quad \forall x \in (x_f(t), L). \quad (35)$$

The determination of  $\phi^d$  is strongly related to the method used to inject the resin in the preform. Assuming a pressure driven infiltration, i.e. a known pressure pushing the resin into the preform, then

$$\phi^d(t) = \Sigma_s^{-1}(\Delta P(t)), \quad (36)$$

where  $\Sigma_s^{-1}$  is the inverse function of  $\Sigma_s$  and  $\Delta P = P_o - P_{atm}$ , being  $P_{atm}$  the atmospheric pressure. This means that at time  $t = 0$   $x_e$  and  $x_f$  suddenly move to

$$x_e(0) = x_f(0) = L \left( 1 - \frac{\phi_r}{\phi^d(0)} \right). \quad (37)$$

In particular, if the applied pressure is constant in time, as we shall assume in the following, then  $\phi^d$  remains constant and, using the conservation of the solid mass and the fact that the right border of the preform is fixed, we have that

$$v_s^d(x, t) = 0 \quad \forall x \in (x_f(t), L), \quad t > 0. \quad (38)$$

Finally, the heat equation (13) simplifies to

$$\rho_s C_s \frac{\partial \theta^d}{\partial t} = k_s \frac{\partial^2 \theta^d}{\partial x^2}. \quad (39)$$

In the wet region  $D^w$  the mechanical problem can not be solved analytically but can be reduced to a single parabolic equation (see [6] for more details)

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x} (\phi Q) - (1 - \phi^d) \dot{x}_f \frac{\partial \phi}{\partial x}, \quad (40)$$

where  $Q$  is given by

$$Q = \frac{K(\phi)}{\mu(\delta, \theta)} \frac{d\Sigma_m(\phi)}{d\phi} \frac{\partial \phi}{\partial x}. \quad (41)$$

The boundary conditions to join to (40) are obtained from (16), (17), (20) and (22) which give

$$\phi(x_e, t) = \phi_r \quad t > 0, \quad (42)$$

$$\phi(x_f, t) = \phi^-(t) \quad t > 0, \quad (43)$$

where  $\phi^-$  is the solution of  $\Sigma_m(\phi^-) = \Delta P(t)$ .

The heat and cure equations rewrite as follows

$$\begin{aligned} \rho_m C_m \left( \frac{\partial \theta}{\partial t} + v_s \frac{\partial \theta}{\partial x} \right) &= -\rho_l C_l Q \frac{\partial \theta}{\partial x} + \\ &\frac{\partial}{\partial x} \left[ k_m \frac{\partial \theta}{\partial x} \right] + (1 - \phi) H_c f_c(\theta, \delta), \end{aligned} \quad (44)$$

$$\frac{\partial \delta}{\partial t} + v_s \frac{\partial \delta}{\partial x} = -\frac{Q}{1 - \phi} \frac{\partial \delta}{\partial x} + f_c(\theta, \delta).$$

Using (16), (19) and assuming continuity of  $C_s$  the boundary conditions on the heat flux (18) rewrites

$$\left[ \left[ k(x_f, t) \frac{\partial \theta}{\partial x}(x_f, t) \right] \right] = 0. \quad (45)$$

From (14) and (15) it is possible to deduce the evolution equations for the free boundaries  $x_e$  and  $x_f$ . From Darcy's law one can deduce the following Stefan type conditions

$$\dot{x}_f = \left[ \frac{\phi}{\phi^d} \frac{1}{1 - \phi} \frac{K}{\mu} \frac{d\Sigma_m}{d\phi} \frac{\partial \phi}{\partial x} \right]_{x_f}, \quad (46)$$

$$\dot{x}_e = (1 - \phi^d) \dot{x}_f - \left[ \frac{K}{\mu} \frac{d\Sigma_m}{d\phi} \frac{\partial \phi}{\partial x} \right]_{x_e}. \quad (47)$$

In conclusion the dynamic of the whole system is governed, in the Eulerian formalism, by a non-linear free boundary problem defined in  $D^w$  coupled with a parabolic equation defined in  $D^d$ . Such a problem is characterized by the presence of two free boundaries which give rise to several mathematical difficulties. In order to reduce them we consider, both for a qualitative and quantitative analysis, a set

of material coordinates  $\xi$  fixed on the solid. As reference configuration we consider the one in which the solid is dry, at rest and at the compression configuration determined by (36)

$$\frac{\partial x}{\partial \xi} = \frac{\phi_o}{\phi}. \quad (48)$$

In this Lagrangian reference, since  $x_e(t)$  corresponds to  $\xi = 0$ , the wet domain becomes  $D^w = \{0 \leq \xi \leq \xi_f(t)\}$  where  $\xi_f(t)$  denotes the solid particle just wet by the infiltration front, i.e. it locates the free boundaries between the wet and the dry domain. We then obtain the following initial boundary value problem

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \frac{\phi^2}{\phi_o} \frac{\partial}{\partial \xi} \left( Z \frac{\partial \phi}{\partial \xi} \right), & 0 < \xi < \xi_f(t), \\ \rho_m C_m \frac{\partial \theta}{\partial t} &= -\frac{\phi}{\phi_o} \rho_l C_l Z \frac{\partial \phi}{\partial \xi} \frac{\partial \theta}{\partial \xi} \\ &\quad + \frac{\phi}{\phi_o} \frac{\partial}{\partial \xi} \left( k_m \frac{\phi}{\phi_o} \frac{\partial \theta}{\partial \xi} \right) + (1 - \phi) H_c f_c(\theta, \delta), & 0 < \xi < \xi_f(t), \\ \frac{\partial \delta}{\partial t} &= -\frac{\phi}{\phi_o (1 - \phi)} Z \frac{\partial \phi}{\partial \xi} \frac{\partial \delta}{\partial \xi} + f_c(\theta, \delta), & 0 < \xi < \xi_f(t), \\ \rho_s C_s \frac{\partial \theta^d}{\partial t} &= k_s \frac{\partial^2 \theta^d}{\partial \xi^2}, & \xi_f(t) < \xi < L, \\ \dot{\xi}_f &= \left( \frac{Z}{1 - \phi} \frac{\partial \phi}{\partial \xi} \right)_{\xi_f}, \\ \phi(0, t) &= \phi_r, \\ \phi(\xi_f(t), t) &= \phi^-, \\ \theta(0, t) &= \theta_{in}, \\ \theta(\xi_f(t), t) &= \theta^d(\xi_f(t), t), \\ \theta^d(L, t) &= \theta_L, \\ \left( k_m \frac{\phi}{\phi_o} \frac{\partial \theta}{\partial \xi} \right)_{\xi_f} &= \left( k_s \frac{\partial \theta^d}{\partial \xi} \right)_{\xi_f}, \\ \delta(0, t) &= \delta_{in}, \end{aligned}$$

where

$$Z = \frac{K(\phi)}{\mu(\delta, \theta)} \frac{\phi}{\phi_o} \frac{d\Sigma_m}{d\phi}, \quad (49)$$

and the initial conditions are  $\theta^d(\xi, 0) = \theta_o^d$  and  $\xi_f(0) = 0$ .

The numerical simulations of the 1D non-isothermal infiltration problem are performed using an implicit finite difference method, in which the time step is chosen so that the infiltration front jumps from node to node. Domain decomposition techniques are used to interface the problems in the wet and dry regions.

Variable	Value	Variable	Value
$\rho_s$	2560 $Kg/m^3$	$H$	$1.54 \cdot 10^8 J/m^3$
$\rho_l$	1100 $kg/m^3$	$c_1$	$3.7833 \cdot 10^5 s^{-1}$
$C_s$	670 $J/Kg$	$c_2$	$6.7833 \cdot 10^5 s^{-1}$
$C_l$	16801 $J/Kg$	$E_1$	54418 $J/mole$
$k_s$	0.168 $W/m^\circ K$	$E_2$	50232 $J/mole$
$k_l$	0.0335 $W/m^\circ K$	$E_\mu$	18000 $J/mole$
$m_c$	0.3	$\bar{\mu}$	$2.78 \cdot 10^{-4} Pa s$
$n_c$	1.7	$c_\mu$	1.5
$\delta_g$	0.1	$d_\mu$	1

Table 1: Numerical values of the physical parameters obtained from [13].

The simulations presented in this section refer to a solid preform having length  $L = 30 \text{ cm}$  and rest solid volume fraction  $\phi_r = 0.5$ . The values of the parameters used are given in [13] and have been reported in Table 1 for sake of completeness.

In the simulations performed the infiltration process is driven by a constant pressure. The different elastic behavior of the wet and dry solid generates a jump of  $\phi$  at the infiltration front, since different compressions in the two regions yield the same stress. The elasticity assumption gives rise to a sudden compression of the porous material to a volume ratio corresponding to the applied pressure. In particular, when the applied pressure is  $0.1 \text{ MPa}$  the corresponding solid volume fraction is  $\phi^d = 0.532$ , corresponding to an initial compression of the 6% of the initial preform length, i.e. from  $300 \text{ mm}$  to  $282 \text{ mm}$ .

Figures (1), (2) and (3) give respectively the evolution of  $\phi$ ,  $\delta$ , and  $\theta$ . In particular, we can note that the resin near  $x_e$  is gelling at  $t = 8 \text{ s}$  which stops the infiltration. However, near  $x_e$  resin has not gelled yet which gives rise to further relaxation. This means that a driving pressure of  $0.1 \text{ MPa}$  is not sufficient to achieve infiltration before resin gels. A higher infiltration pressure should be used, in spite of the fact that this will generate a stronger compression as a counterpart.

Figure (4) shows the evolution in time of  $x_e$  (continuous line) and  $x_f$  (dotted line). As already stated, in this case the preform is initially compressed to  $x_f(0) = x_e(0) = 1.82 \text{ cm}$ . We point out that at early times, when curing is not yet important, interfaces move as  $\sqrt{t}$ . This behaviour was predicted for a much simpler model (isothermal and same constitutive equations) in [7]. In this paper using a Lagrangian formulation of the problem, the existence and uniqueness of a self-similar solution has been proved.

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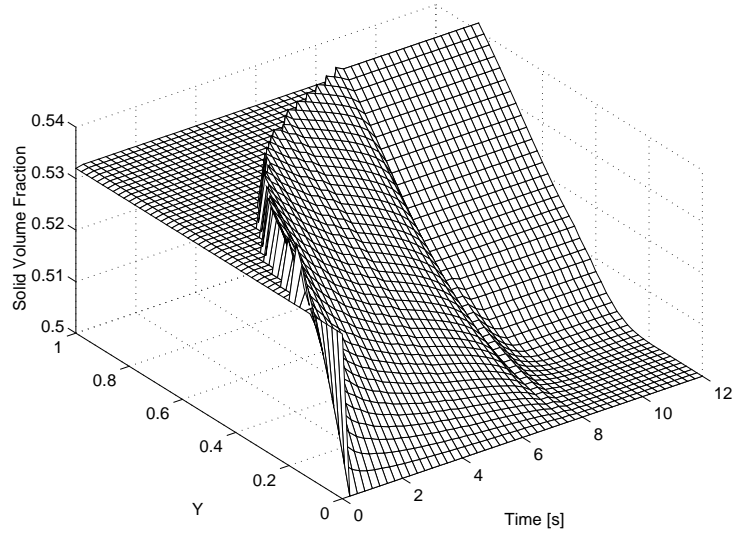


Figure 1: Solid volume fraction  $\phi(Y, t)$ .  $Y = \xi/L$  normalized Lagrangian coordinate. Applied pressure  $0.1 \text{ MPa}$ .

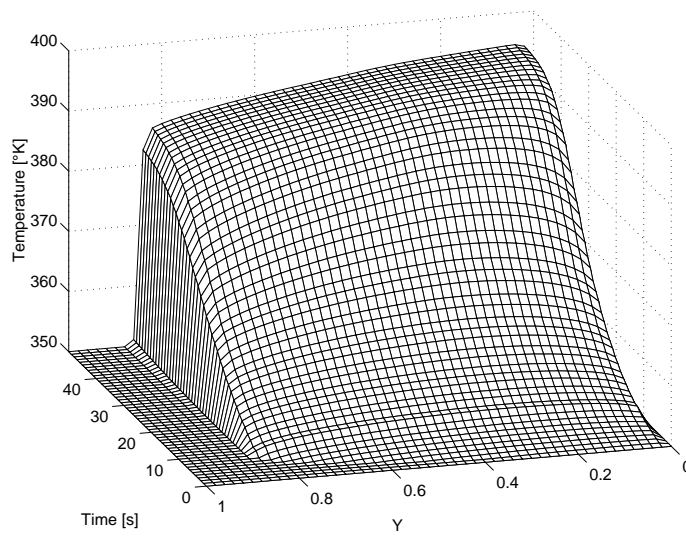


Figure 2: Temperature  $\theta(Y, t)$ .  $Y = \xi/L$  normalized Lagrangian coordinate. Applied pressure  $0.1 \text{ MPa}$ .

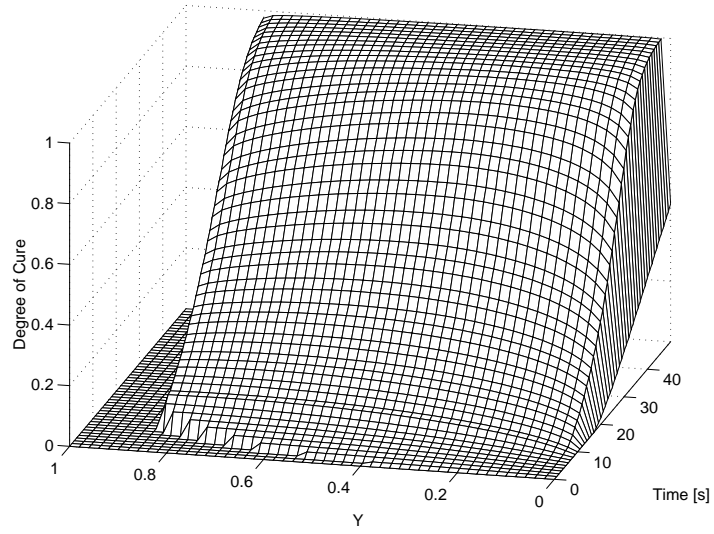


Figure 3: Degree of resin cure  $\delta(Y, t)$ .  $Y = \xi/L$  normalized Lagrangian coordinate. Applied pressure  $0.1 \text{ MPa}$ .

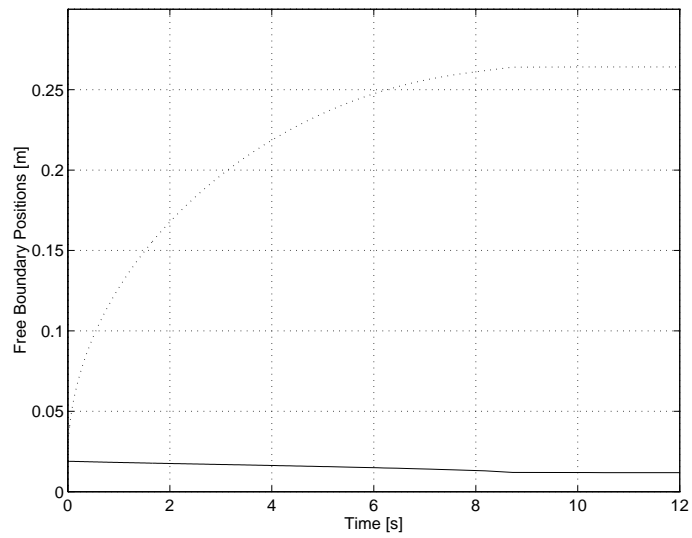


Figure 4: Positions of the free boundaries. The dotted line corresponds to  $x_f(t)$ . The continuous line corresponds to  $x_e(t)$ . The pressure driving the flow is  $0.1 \text{ MPa}$ .