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An algebraic theory suited for tropical mathematics

Many algebraic theories involve the study of a set T with fragmented structure which can be understood better by embedding T in a larger set A endowed with more structure. Classical examples include the homogeneous components of a graded algebra. In the direction of tropical mathematics, the max-plus algebra and related tropical structures were embedded by Izhakian into semirings which are more manageable, and the same can be said for hypergroups and fuzzy rings.

On the other hand, in mathematical theories involving semirings, one often is challenged by the lack of negation when trying to formulate the tropical versions of classical algebraic concepts for which the negative is a crucial ingredient. Developing an idea of Gaubert in his doctoral dissertation and brought to fruition by Akian, Gaubert, and Guterman, we study **triples** $(A, T, (-))$ with negation maps, in the context of universal algebra, showing how these unify the more viable (super)tropical versions, as well as hypergroup theory and fuzzy rings, thereby "explaining" similarities in the various theories. Special attention is paid to **meta-tangible** triples, defined by the property that $a + b \in T$ for all $a, b \in T$ for which $b \neq (-)a$.

Furthermore, equality on T generalizes to a relation $\preceq$ on A which plays a key

structural role, yielding a **system**. Their algebraic theory includes all the main tropical examples and many others, but is rich enough to facilitate computations and provide a host of structural results. Systems can be "fundamental," insofar as they provide the underlying structure, which then is studied via classical structure theory, as well as linear algebra (in ongoing research with Akian and Gaubert) and through representation theory via "module" systems (in ongoing work with Jun and Mincheva, paralleling research of Connes and Consani).

This approach enables one to view the tropicalization functor as a morphism, thereby indicating tropical analogs of such classical algebraic structures as Grassmann algebras, Lie algebras, Lie superalgebras, Poisson algebras, and Hopf algebras.