

SEMINARIO DI GEOMETRIA

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Conformal Strings

Möbius geometry of space curves was developed mostly in the first decades of the past century. In more recent times, the subject has been reconsidered in the context of the regularization of the Kepler problem. However, the reasons which have determined the renewed interest in this classical topic must be traced in the theory of integrable systems and in the topology of knots : local motions of curves associated to geometric Poisson brackets, finite or infinite-dimensional integrable Hamiltonian systems, hierarchies of integrable evolution equations and Möbius energy of knots. In this talk we investigate closed trajectories of the simplest Möbius-invariant variational problem for space curves, defined by the integral of the conformal arc-element. The variational problem is non-degenerate and admits a description in terms of a contact Hamiltonian system on the momentum space of the functional. The action of the Möbius group on the momentum space is Hamiltonian and co-isotropic and the system is collectively completely integrable. Consequently, the Hamiltonian vector field can be linearized on the fibers of the momentum map and its trajectories can be found by quadratures. Another interesting feature is that the critical curves of the functional evolve by conformal congruences with respect to an invariant curve flow. We will focus on closed critical curves with non-constant conformal curvatures, conformal strings for brevity. Our aim is to provide a rather complete description of the geometric behavior of such curves and to develop numerical and symbolical routines to compute the string form the knowledge of its quantum numbers.