An approach to computing the cactus rank of a cubic form

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Given a homogeneous polynomial F of degree d in $S := K[x_0, \ldots, x_n]$, its *rank* is the minimum number r of linear forms $L_1, \ldots, L_r$ such that

$$F = {L_1}^d + \cdots + {L_r}^d.$$

Consider the polynomial ring $T := K[y_0, \ldots, y_n]$ acting on S by differentiation and let $F^\perp := \{g \in T : g(f) = 0\}$. The rank of F is also the minimal degree of a smooth scheme apolar to F, i.e. a scheme Γ whose homogeneous ideal I_Γ is contained in $F^\perp$. We define the *cactus rank* of F as the minimal degree of *any* scheme apolar to F (not necessarily smooth).

Bernardi and Ranestad proved that the cactus rank of a general cubic form F is at most $2n + 2$ and conjectured that this upper bound is attained for $n \geq 8$. In a joint work with these authors, we give an approach to computing the cactus rank of a general cubic form and apply this technique to cases $6 \leq n \leq 10$.