

SEMINARIO DI GEOMETRIA

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Hyperbolic polynomials and multiparameter real analytic perturbation theory

Joint work with L. Paunescu (Sydney). Let $P(x, z) = z^d + \sum_{i=1}^d a_i(x)z^{d-i}$ be a polynomial, where a_i are real analytic functions in an open subset U of \mathbb{R}^n . If for any $x \in U$ the polynomial $z \mapsto P(x, z)$ has only real roots, then we can write those roots as locally Lipschitz functions of x . Moreover, there exists a modification (a locally finite composition of blowing-ups with smooth centers) $\sigma : W \rightarrow U$ such that the roots of the corresponding polynomial $\tilde{P}(w, z) = P(\sigma(w), z)$, $w \in W$, can be written locally as analytic functions of w . Let $A(x)$, $x \in U$ be an analytic family of symmetric matrices, where U is open in \mathbb{R}^n . Then there exists a modification $\sigma : W \rightarrow U$, such the corresponding family $\tilde{A}(w) = A(\sigma(w))$ can be locally diagonalized analytically (i.e. we can choose locally a basis of eigenvectors in an analytic way). This generalizes the Rellich's well known theorem (1937) for one parameter families. Similarly for an analytic family $A(x)$, $x \in U$ of antisymmetric matrices there exists a modification σ such that we can find locally a basis of proper subspaces in an analytic way.