SEMINARIO DI GEOMETRIA

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Politecnico di Torino,
Dipartimento di Scienze Matematiche,
AULA BUZANO

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Analogues of coverings and the fundamental group in geometry of partial differential equations

It is known that, using jet bundles, one can regard partial differential equations (PDEs) as geometric objects. Namely, a PDE can be regarded as a manifold with a distribution such that solutions of the PDE correspond to certain integral submanifolds of the distribution. This allows one to study (nonlinear) PDEs by means of methods of differential geometry.

Recall that fundamental groups are an important invariant for topological spaces. In this talk, we introduce an analogue of fundamental groups for PDEs. However, the fundamental group of a PDE is not a group, but a certain system of Lie algebras, which we call fundamental Lie algebras. Fundamental Lie algebras are new geometric invariants for PDEs and are closely related to integrability properties of PDEs, where integrability is understood in the sense of soliton theory.

In particular, using fundamental Lie algebras, we obtain necessary conditions for integrability and necessary conditions for existence of Backlund transformations for nonlinear $(1 + 1)$-dimensional evolution PDEs. In the structure of fundamental Lie algebras for integrable $(1 + 1)$-dimensional PDEs, one finds infinite-dimensional subalgebras of Kac-Moody algebras and infinite-dimensional Lie algebras of certain matrix-valued functions on some algebraic curves. To develop this theory, we use a generalization of the Wahlquist-Estabrook prolongation method and the theory of coverings of PDEs invented by A. Vinogradov and I. Krasilshchik. Some results of this talk have been obtained in joint works with G. Manno (Politecnico di Torino).