Linearized field equations in Cauchy-Riemann geometry

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The lecture is devoted to the study of linearized gravitational field equations in Cauchy-Riemann geometry. It comprises two parts, the first of which focuses on the study of the pseudo-Einstein equation $R_{1\bar{1}} = 0$ on the 3-dimensional Heisenberg group $\mathbb{H}_1 = \mathbb{C} \times \mathbb{R}$. Here one considers first order perturbations $\theta_\epsilon = \theta_0 + \epsilon\,\theta$, $\epsilon << 1$, and linearizes the pseudo-Einstein equation about θ_0 (the canonical Tanaka-Webster flat contact form on $\mathbb{H}_1$ thought of as a strictly pseudoconvex CR manifold). If $\theta = e^{2u}\theta_0$ the linearized pseudo-Einstein equation is $\Delta_b u - 4|Lu|^2 = 0$ where Δ_b is the sublaplacian of $(\mathbb{H}_1, \theta_0)$ and $\overline{L}$ is the Lewy operator. We solve the linearized pseudo-Einstein equation on a bounded domain $\Omega \subset \mathbb{H}_1$ by applying subelliptic theory i.e. existence and regularity results for weak subelliptic harmonic maps. Also we determine a solution u to the linearized pseudo-Einstein equation, possessing Heisenberg spherical symmetry, and such that $u(x) \to -\infty$ as $|x| \to +\infty$.

The second part of the lecture starts from the total space $\mathfrak{M} \approx \mathbb{H}_1 \times S^1$ of the canonical circle bundle over $\mathbb{H}_1$ endowed with the Lorentzian metric F_{θ_0} on $\mathfrak{M}$ (Fefferman's metric) associated to θ_0. The matter and energy content of $\mathfrak{M}$ is described by the energy-momentum tensor $\mathring{T}_{\mu\nu}$ (the trace-less Ricci tensor of F_{θ_0}) as an effect of the non flat nature of Fefferman's metric F_{θ_0}. We study the gravitational field equations $R_{\mu\nu} - (1/2)\, R\, g_{\mu\nu} = \mathring{T}_{\mu\nu}$ on $\mathfrak{M}$. We consider the first order perturbation $g = F_{\theta_0} + \epsilon\, h$, $\epsilon << 1$, and linearize the field equations about F_{θ_0}. We determine a Lorentzian metric g on $\mathfrak{M}$ which solves the linearized field equations corresponding to a diagonal perturbation h.

References

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[2] E. Barletta & S. Dragomir & H. Jacobowitz, *Gravitational field equations on Fefferman space-times*, to appear in Complex Analysis and Operator Theory, 2017.

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