Maximal rank curves and cohoimology

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Let $R := K[x,y,z,t]$, K algebraically closed field of characteristic zero. Let M be a finite length graded R-module. From Liaison Theory we know that there are curves $C \subseteq \mathbb{P}^3$ whose *Hartshorne-Rao module* $M_C := \bigoplus_{j\in\mathbb{Z}} H^1(\mathcal{I}_C(j))$ is isomorphic to $M(-h)$, provided h is sufficiently large. For any fixed M the curves C such that $M_C \cong M(-h)$ for some $h \in \mathbb{Z}$ form a set $\mathbb{B}(M)$, the *biliaison class* of M. A general problem is to relate the algebraic and/or homological properties o M with the geometry of the curves in $\mathbb{B}(M)$.
The geometric properties we consider are:

(i) the "lifting property" $s(C) = \sigma(C)$, where $s(C)$ is the least degree of a surface containing C and $\sigma(C)$ is the least degree of a planar curve containing a general plane section of C;

(ii) maximal rank: C has *maximal rank* if $H^1(\mathcal{I}_C(j)) = 0$ for $j \geq s(C)$.

We describe some results from a joint paper with R.M. Miró-Roig, concerning existence conditions and constructions of maximal rank curves in $\mathbb{B}(M)$ (with or without the lifting property) and the *Rao function* $\rho_C(j) := h^1(\mathcal{I}_C(j))$ for a maximal rank C. For example:

Let $M \neq 0$ be a finite length graded R-module. Let q be the largest degree of a non-zero element of M, and b the least degree of a second syzygy of M. Assume $b \geq q+1$.
Then $\mathbb{B}(M)$ contains maximal rank curves.
Moreover $\mathbb{B}(M)$ contains also maximal rank curves C with $s(C) = \sigma(C)$ if and only if the map $M_{q-1} \to M_q$ induced by a general linear form is injective.

We conclude with an existence problem for certain maximal rank curves, motivated by a recent result on "curves with large cohomology", which are very far from being of maximal rank (work in progress with G. Paxia).