A formalism for equivariant Schubert calculus. (English summary)

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Equivariant Schubert calculus, in the context of this article, deals with the structure of the equivariant cohomology of the Grassmannian $\text{Gr}(r, \mathbb{C}^n)$ parametrizing $r$-dimensional quotients of $\mathbb{C}^n$. Taking an appropriate limit, one also considers the cohomology of the infinite Grassmannian $\text{Gr}(r, \mathbb{C}^\infty)$. The starting point for the formalism developed here is the observation that the (ordinary) cohomology ring may be identified with an algebra of operators on the $r$th exterior power of a polynomial ring, $\bigwedge^r \mathbb{Z}[T]$. This point of view was used by Gatto and Santiago to give new proofs of the Pieri rule and Giambelli formula [L. Gatto and T. Santiago, Canad. Math. Bull. 52 (2009), no. 2, 200–212; MR2512308 (2010c:14059)].

In previous work (with A. Thorup), the present author extended these ideas to model the cohomology of a Grassmann bundle over an arbitrary base scheme [Indiana Univ. Math. J. 58 (2009), no. 1, 283–300; MR2504412 (2010f:14058)]. This more general setting develops a Schubert calculus in $\bigwedge^r A[T]$, for an arbitrary (commutative) ring $A$.

The present article interprets the above results for equivariant cohomology. Two key features stand out in comparison with other treatments of equivariant Schubert calculus:

1. The “Giambelli formula” expresses Schubert classes as unshifted factorial Schur functions $s_\lambda(T_1, \ldots, T_r|y)$; previous such determinantal formulas require shifts in the indices of the torus variables $y_1, y_2, \ldots$ [see, e.g., L. C. Mihalcea, Trans. Amer. Math. Soc. 360 (2008), no. 5, 2285–2301; MR2373314 (2009e:14099)].

2. The author gives a full equivariant Pieri formula expressing the product of an arbitrary Schubert class by a special class, not merely the Chevalley-Monk formula for product by a divisor. Such formulas were also given by S. Robinson [J. Algebra 249 (2002), no. 1, 38–58; MR1887984 (2003b:14065)] and in the doctoral thesis of Santiago [Schubert calculus on a Grassmann algebra, Politec. Torino, 2006].

Reviewed by David E. Anderson

References


*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

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