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Schubert calculus and equivariant cohomology of Grassmannians. (English summary)

From the introduction: “During the last years there has appeared a rich literature on the equivariant cohomology ring of Grassmannians. We recommend [W. Fulton, “Equivariant cohomology in algebraic geometry”, Eilenberg lectures (notes by D. Anderson), Columbia Univ., 2007, available at www.math.lsa.umich.edu/~dandersn/eilenberg/index.html] for references to the literature as well as for a presentation of equivariant cohomology both from a geometric and algebraic point of view. In this article, we give an alternative description of equivariant cohomology where the basic results of equivariant Schubert calculus, the basis theorem, Pieri’s formula and Giambelli’s can be obtained from the corresponding results of the general framework by a change of basis.

“More precisely, we gave in [D. Laksov and A. Thorup, Indiana Univ. Math. J. 58 (2009), no. 1, 283–300; MR2504412 (2010f:14058)] a general formalism of Schubert calculus for Grassmannians consisting of a factorization algebra \( \text{Fact}_A^l(p) \) of a polynomial \( p(T) \) with coefficients in a ring \( A \), and a structure as an \( \text{Fact}_A^l(p) \)-modules on the exterior power \( \bigwedge^l_A A[T]/(p) \). In this article, we show that when \( p(T) \) is a general factorial power, that is,

\[
p(T) := (T|y)^n = (T - y_1) \cdots (T - y_n)
\]

with \( y_1, \ldots, y_n \) in \( A \), we obtain the equivariant cohomology of Grassmannians from the general formalism by replacing the basis \( 1, \xi, \ldots, \xi^{n-1} \) of \( A[\xi] := A[T]/(p) \), where \( \xi \) is the class of \( T \), by the general factorial powers \( (\xi|y)^0, (\xi|y)^1, \ldots, (\xi|y)^{n-1} \). The reader should also consult [W. Fulton, op. cit.] for the correspondence between the classical and equivariant theories. In particular, Fulton explains in lecture 7 of the notes how the equivariant Giambelli formula for Grassmannians amount to a degeneracy formula [G. Kempf and D. Laksov, Acta Math. 132 (1974), 153–162; MR0338006 (49 #2773)] in algebraic geometry.

“Our work builds upon earlier results. In [D. Laksov and A. Thorup, Indiana Univ. Math. J. 56 (2007), no. 2, 825–845; MR2317547 (2008d:14089)], we gave a formalism where the ring of symmetric functions \( A[T_1, \ldots, T_n]^{\text{sym}} \) in \( n \) variables operates on the exterior power \( \bigwedge^n_A A[T] \). We proved that even in this generality there exists a Schubert calculus having a basis theorem, a Pieri formula and a Giambelli formula. In [D. Laksov, “A formalism of equivariant Schubert calculus”, preprint, 2006; per bibl.], we saw that we from this formalism obtain a corresponding equivariant Schubert calculus by replacing the basis \( 1, T, T^2, \ldots \) of \( A[T] \) by the basis \( (T|y)^0, (T|y)^1, (T|y)^2, \ldots \) of general factorial powers. In this article, we specialize the equivariant Schubert calculus of [D. Laksov, op. cit.] to an equivariant Schubert calculus for Grassmannians. In this way, we obtain the Pieri and the Giambelli formulae in equivariant cohomology by specialization from the corresponding results in [D. Laksov, op. cit.].
“In order to show that our formalism reflects the geometry of Grassmannians we must prove that the expressions given by the equivariant Giambelli formula correspond to the classes in equivariant cohomology given by Schubert schemes in the Grassmannian. This we do by relating our theory to the formalism for equivariant cohomology of A. Knutson and T. C. Tao [Duke Math. J. 119 (2003), no. 2, 221–260; MR1997946 (2006a:14088)]. We note that our approach has several advantages. In [A. Knutson and T. C. Tao, op. cit.] there are two proofs of the existence of Schubert classes, one topological and one combinatorial. Both are ‘top down’, that is, they start from the most equivariant case. In our theory, the existence is part of the general framework of factorial Schur functions and requires no separate proof. Our approach is also computationally efficient.

“We note that we obtain the full Pieri formula, and not only the Chevalley formula for divisors for various forms of the latter formula. It should also be pointed out that our equivariant Giambelli formula is an unshifted version of that of L. C. Mihalcea [Trans. Amer. Math. Soc. 360 (2008), no. 5, 2285–2301; MR2373314 (2009e:14099)], that is, using only general factorial powers and not their shifted counterparts.”

References

14. B. Kostant, S. Kumar, The nil Hecke ring and cohomology of $G/P$ for a Kac–Moody group


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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