

## Abstracts of the Workshop

### “Algorithms and dynamics over networks”

**Dipartimento di Matematica, Politecnico di Torino, July 14-15 2010**

Saverio Bolognani

*Distributed quasi-Newton method and its application to the optimal reactive power flow problem*

We consider a distributed system of  $N$  agents, on which we define a quadratic optimization problem subject to a linear equality constraint. We assume that the nodes can estimate the gradient of the cost function by measuring the steady state response of the system. Even if the cost function cannot be decoupled into individual terms for the agents, and the linear constraint involves the whole system state, we are able to design a distributed, gradient-driven, algorithm, for the solution of the optimization problem. This algorithm belongs to the class of quasi-Newton methods and requires minimal knowledge of the system to behave fairly well. We proved finite time convergence of the algorithm in its centralized version, and we designed its distributed implementation in the case in which a communication graph is given. In this latter case, the tool of average consensus results to be fundamental for the distribution of the algorithm. As a testbed for the proposed method, we consider the problem of optimal distributed reactive power compensation in smart microgrids.

Ming Cao

*Cluster Synchronization Algorithms in Diffusively Coupled Networks*

We present two approaches for achieving cluster synchronization in diffusively coupled networks. In contrast to the widely studied synchronization behavior, where the states of all the coupled nodes converge to the same value asymptotically, in the cluster synchronization problem discussed in this talk, we require the interconnected nodes to evolve into several clusters and each node only to synchronize within its cluster. The first approach is to add a constant forcing to the dynamics of each node that are determined by positive diffusive couplings; and the other is to introduce both positive and negative couplings between the nodes. Some sufficient and/ or necessary conditions are constructed to guarantee  $n$ -cluster synchronization behavior.

Giacomo Como

*Persistent disagreement in social networks*

Disagreement among individuals in a society is the rule; agreement is the rare exception. How can disagreement of this sort persist for so long? Existing models of opinion dynamics typically lead to consensus provided that communication takes place over a strongly connected network. We analyze a stochastic gossip model of communication combined with the assumption that there are some “stubborn” agents in the network who never change their opinions. We show that the presence of these stubborn agents leads to persistent disagreement among the rest of the society. The opinions of the agents keep on fluctuating ergodically, and approach an asymptotic distribution whose moments can be characterized

in terms of the hitting probabilities of a coalescing random walk on the network. An analysis of such stationary opinion distribution reveals that, for most random networks of interest, the amplitude of oscillations and mutual disagreement does not vanish in the limit of large population.

Claudio De Persis

*Control of one-dimensional guided formations using binary information*

Motivated by applications in intelligent highway systems, we study the problem of guiding mobile agents in a one-dimensional formation to their desired relative positions. Only binary information is used which is communicated from a guidance system that monitors in real time the agents' motions. The desired relative positions are defined by the given distance constraints between the agents under which the overall formation is rigid in shape and thus admits locally a unique realization. It is shown that even when the guidance system can only transmit one bit of information to each agent, it is still possible to design control laws to guide the agents to their desired positions.

Jean-Charles Delvenne

*Consensus, partial consensus, and communities*

Consider a large number of agents following a linear time invariant dynamics leading to consensus. In many practical applications, the communication graph comprises bottlenecks: some groups of agents, although communicating closely between them, interact weakly with the rest of the network. A partial consensus is quickly reached within such a group, long before the global consensus is reached. The design of algorithms to detect 'clusters' or 'communities' (i.e., groups of nodes very connected between them but barely connected to the rest of the network) in large social, biological or technological networks has been very popular in the last decade. We show how the phenomenon of partial consensus can be used to unify some of those algorithms and propose new community detection techniques.

Sophie Fosson

*Deconvolution of quantized-input linear systems*

The deconvolution problem, ubiquitous in many scientific and technological areas, consists in recovering the unknown input of a linear system from a noisy version of the output. The case of a system with quantized input is particularly interesting since it can be undertaken using decoding techniques instead of classical estimation methods. In this framework, we introduce a low-complexity decoding algorithm, derived from the well-known BCJR, to perform deconvolution of a binary input signal and we analyse its performance through the Ergodic Theory of Markov Processes, with some unexpected implications.

Paolo Frasca

*Broadcast gossip averaging algorithms: interference and asymptotical error*

In this talk we present and study two related iterative randomized algorithms for distributed computation of averages. In these algorithms, named Broadcast Gossip algorithms, at each iteration some randomly selected nodes broadcast their states to their neighbors. Both algorithms are proved to converge and their performance is evaluated in terms of rate of convergence and asymptotical error. A thorough mean

square analysis is performed, under the assumption of suitable symmetries in the network topology. We highlight the role of the graph topology and the effects of the design parameters on the performance, focusing on the behavior for large networks. In particular, we prove that on sparse graphs with bounded degree, both algorithms are asymptotically unbiased, in the sense that the asymptotical errors go to zero as the network grows large.

Floriana Gargiulo,

*Groups and opinion co-evolution on social networks*

Many sociological studies underlined the fundamental role of group structures on the formation of opinion in large communities. Moreover the recent availability of data on WEB 2.0 social networks and the statistical treatment of these datasets provided some interesting information on the structure and on the classification of the groups. Motivated by these results we studied a theoretical model where the group structures on a social network are co-evolved with a simple opinion dynamics contact process. Our model exhibits two interesting results: first of all the presence of group dynamics can change the convergence properties of opinion dynamics. Secondly opinion dynamics processes, coupled with network dynamics, can generate different scenarios of group hierarchy according to the parameter choice: starting from an homogeneous group size distribution, the opinion dynamics process generates, for a large range of the parameters, macroscopic groups.

Federica Garin

*Distributed averaging on digital noisy networks*

We propose a class of distributed algorithms for computing arithmetic averages (average consensus) over networks of agents connected through digital noisy broadcast channels. These algorithms do not require the agents to have any knowledge about the global network structure or size. Almost sure convergence to state agreement is proved, and the communication and computational complexities of the algorithms are analyzed.

Enrico Lovisari

*A resistance-based approach to consensus algorithm performance analysis*

We study the well known linear consensus algorithm by means of a LQ-type performance cost. We want to understand how the communication topology influences this algorithm. In order to do this, we recall the analogy between Markov Chains and electrical resistive networks. By exploiting this analogy, we are able to rewrite the performance cost as the average effective resistance on a suitable network. We use this result to show that if the communication graph fulfills some local properties, then its behavior can be approximated with that of a suitable grid, over which the behavior of the cost is known.

Chiara Ravazzi

*Matrix completion via regular sampling*

We are interested in recovering an unknown matrix observing a fraction of its entries. This is known as the matrix completion problem, and comes up in a great number of applications in machine learning,

statistics and collaborative filtering. A famous instance is the celebrated Netflix Prize. In general, accurate recovery is impossible; but the knowledge that the unknown matrix has low rank makes the search for solutions meaningful. Recently the first non-trivial theoretical results for the problem have been obtained, assuming that the observed entries are sampled uniformly at random. Most of the literature has focused on improving the estimate on the number of revealed entries requested to complete the matrix --at the cost of stronger hypothesis on the matrix being measured--, or on devising faster and better algorithms for the actual recovery. Here, inspired by techniques coming from coding theory, we are interested in improving previous bounds by using a regular sampling model.

Andrea Tosin

*A multiscale look at pedestrian flows by time-evolving measures*

In this talk we present an approach stemming from measure and optimal transport theory for modeling pedestrian flows under a multiscale point of view. The microscopic granularity of the individuals is incorporated in the macroscopic flow of the crowd by means of mass measures featuring an absolutely continuous and a singular part. This way a dynamics coupling both scales is obtained, ruled by the continuity equation. Time and space discretization of the resulting equations, along with related analytical issues, will be discussed.