Distributed problems over networks

Communication constraints in optimization and control

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Networks are ubiquitous

- Computer networks (Internet, WWW)
- Social networks (LinkedIn, Facebook, Erdős numbers)
- Sensor networks
- Animal groups (flocks, swarms, schools)
- Groups of unmanned vehicles - robots

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A novel science of networks

Goals:

- Understanding collective phenomena (in nature)
- Design of distributed algorithms (in engineering)

Tools:

- Graph theory
- Information theory
- Optimization and Control theory

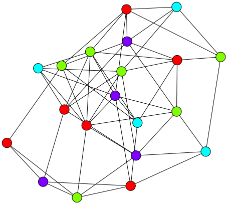
General framework

In general, an optimization problem is

$$\min_{\theta \in \Theta} f(\theta).$$

We have a graph $\mathcal{G}$

- nodes are "agents";
- edges are available communication links.



$$f(\theta) = \sum_{i=1}^{N} f_i(\theta).$$

Each function f_i is local to agent i.

Special cases

- Distributed decision/computation:
  e.g. average consensus
- Optimal deployment - Coverage control
- Flocking - Formation control

Consensus with digital communication

Problem:
Every node knows one real value, and wants to compute a function (e.g., the average) of such numbers.

Constraint:
Communication across the graph edges is digital (finite precision, errors, noise)

Most recent result (2009):
The average can be computed up to any given precision on a digital network, in a time which is poly-logarithmic in the precision.

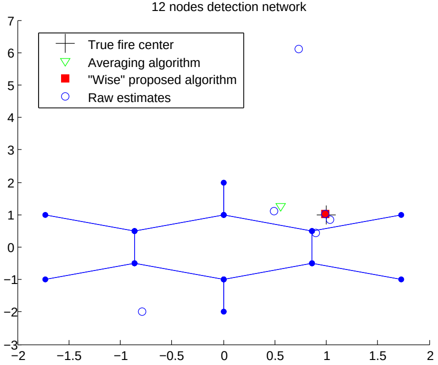
Joint work with F. Fagnani (PoliTo), R. Carli (UCSB), G. Como (MIT), F. Garin and S. Zampieri (UniPadova).

Distributed estimation

Application: Distributed fire detection

How to locate a fire center? Using a network of sensors which

1. take temperature measurements;
2. communicate with neighbors to make a first estimate;
3. iteratively agree on a common estimate.

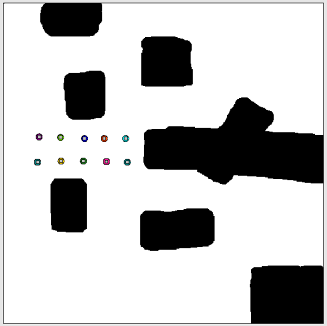


Deployment of self-propelled robots

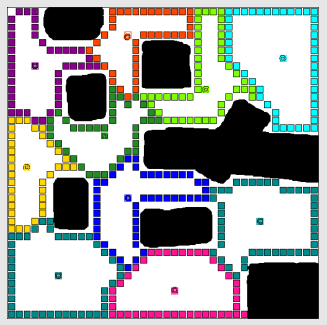
- complex environment,
- peer-to-peer communication with neighbors,
- optimization of the performance function

$$\mathcal{H}_{\text{centroid}}(v) = \sum_{i=1}^{N} \int_{v_i} f(\|q - \text{Cd}(v_i)\|)\phi(q)dq.$$

Initial positions



Final positions

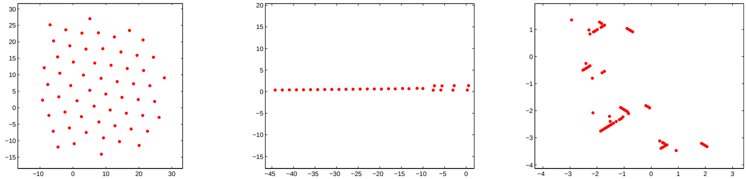


JOINT WORK WITH F. BULLO, R. CARLI, J. DURHAM (UNIVERSITY OF CALIFORNIA, SANTA BARBARA)

Flocking of animal/vehicle groups

- Global or local (agent-wise) cohesion-repulsion potentials; and
- suitable information flow (e.g. restricted frontal view)

$\Longrightarrow$ structured group behaviors.



Joint work with E. Cristiani, B. Piccoli (IAC-CNR)

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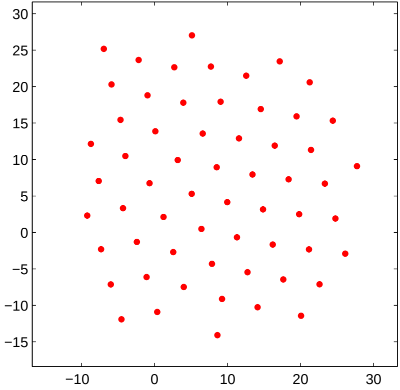

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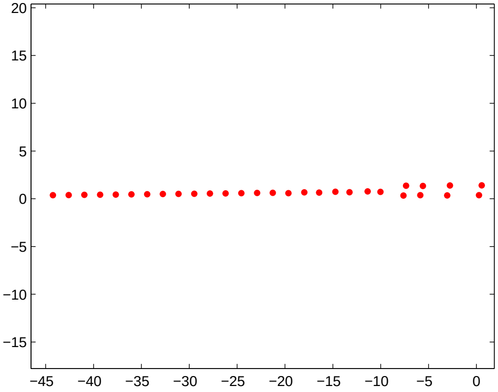

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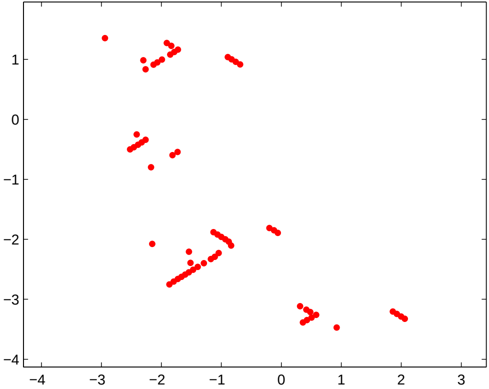

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