TWO-SCALE CONVERGENCE METHOD AND SCATTERING PROBLEMS

These works ([JMS05] and [SUZDAL02]) deal with the stady-state wave equation in $\mathbf{R}^3$, deffracted by an obstacle made of a non homogeneous medium and located in a bounded domain of $\mathbf{R}^3$

$$-\frac{\partial}{\partial x_i}\left(a_{ij}^{\varepsilon}(x)\frac{\partial u_\varepsilon}{\partial x_j}\right) - \omega^2 u_\varepsilon = f \qquad \text{in } \mathbf{R}^3 \ .$$

The Sommerfeld radiation conditions at infinity in the integral form is discussed

$$u_\varepsilon^{\pm} = \frac{1}{4\pi}\int_{|y|=R}\left(\frac{\partial u_\varepsilon^{\pm}}{\partial n}\left(\frac{\mathrm{e}^{\pm \mathrm{i}\omega r}}{r}\right) - u_\varepsilon^{\pm}\frac{\partial}{\partial n}\left(\frac{\mathrm{e}^{\pm \mathrm{i}\omega r}}{r}\right)\right)\mathrm{d}\,S \ .$$

The non homogenities of the medium, depend on a parameter $\varepsilon > 0$, which tends to zero. We suppose, as in the homogenization process, that the solution u_ε of the stady-state wave equation, for ε fixed, tends to a solution of an analogous problem when ε goes to zero.

The main method is outlined to solving the reduced wave equation. This method consists of a reduction to a boundary value problem, which immediately gives the scattering frequencies as the singularities of the analitic continuation of the solution, with respect to the parameter ω (where the dependence in time is of the form $\mathrm{e}^{-\mathrm{i}\omega t}$).

The two-scale convergence method is presented and it gives us the convergence of u_ε and of the gradient of u_ε

$$\nabla u_\varepsilon \qquad \overset{two-scale}{\rightarrow} \qquad \nabla_x u + \nabla_y u_1 \ .$$

Moreover we apply the two-scale convergence method to study the limit of stady-state wave equation depending from the parameter ε in order to obtain

$$\int_\Omega\int_Y a_{ij}(x,y)\left(\frac{\partial u(x)}{\partial x_j} + \frac{\partial u_1(x,y)}{\partial y_j}\right)\left(\frac{\partial \varphi(x)}{\partial x_i} + \frac{\partial \varphi_1(x,y)}{\partial y_i}\right)\mathrm{d}\,x\,\mathrm{d}\,y =$$

$$= \int_\Omega f(x)\varphi(x)\,\mathrm{d}\,x$$

where $\Omega = \{x \in \mathbf{R}^3 : |x| < r\}$ with r sufficiently large. The limit process is considered for the scattering frequencies and functions too.