A note on Bisexual Galton-Watson Branching Processes in Random Environments

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Abstract

A bisexual Galton-Watson branching process is a two–type branching model, in which matings in one generation give rise to random numbers of both males and females in the next. The mating function describes how many mating units are formed from given numbers of males and females. In this paper we consider the case that the distributions of the random numbers of males and females produced by the mating units depend on some fertility parameters evolving randomly in time. By means of a main stochastic comparison result, we show that the total population increases, in some stochastic sense, as the positive dependence between the fertility indexes increases. Simple examples of applications of this result are provided, together with other similar results for a different model of population growth.

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